Matching of the continuous gravitational wave in an all sky search

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Abstract
We investigate the matching of continuous gravitational wave (CGW) signals in an all sky search with reference to Earth based laser interferometric detectors. We consider the source location as the parameters of the signal manifold and templates corresponding to different source locations. It has been found that the matching of signals from locations in the sky that differ in their co-latitude and longitude by $\pi$ radians decreases with source frequency. We have also made an analysis with the other parameters affecting the symmetries. We observe that it may not be relevant to take care of the symmetries in the sky locations for the search of CGW from the output of LIGO-I, GEO600 and TAMA detectors.

Keywords: gravitational wave – methods: data analysis – pulsars: general.

1 Introduction

The first generation of kilometer-scale gravitational wave (GW) laser interferometric detectors with sensitivity in the frequency band 10 Hz to few kHz and ultra cryogenic bar detectors sensitive at frequencies around 1 kHz will start collecting data soon. The TAMA

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300 (Tsubona 1995) has already done the first large scale data acquisition (Tagoshi et al. 2001), while LIGO (Abramovici et al. 1992) and GEO600 (Danzmann 1995) have recently carried out their first science observations. VIRGO (Bradaschia et al. 1991) may become operational in couple of years. Also, an eighty meter research interferometer ACIGA (McCleland et al. 2000) near Perth, Australia is under construction, hoping that it may be possible to extend it to multi-kilometer scale in the future.

At present, majority of searches are focussed in the detection of *chirp* and burst signals. However interest for the search of continuous gravitational wave (CGW) signals from the output of detectors is growing (Jaranowski, et. al 1998, Jaranowski & Królak 1999, 2000; Astone et al. 2002; Brady et. al. 1998, Brady & Creighton 2000) due to the possibility of enhancing the signal-to-noise ratio (S/N) by square root of observation time $\sqrt{T_{\text{obs}}}$. An optimistic estimate suggests that the Earth based laser interferometric detectors may detect such signals with an observation time of 1-yr.

The strength of the CGW largely depend on the degree of long-lived asymmetry in the source. There are several mechanism for producing such an asymmetry (Pandharipande et al. 1976; Bonazzola & Gourgoulhon 1996; Zimmermann & Szednits 1979; Zimmermann 1980). The estimates of the asymmetry in neutron stars shows that the amplitude of CGW may be $\leq 10^{-25}$. Hence, long integration will be required to get the signature of signal. But this in turn induces several other problems viz. Doppler modulation and non-stationarity of the detector noise. Consequently data analysis becomes more harder. However, Doppler modulation will provide the information of position of the source in the sky.

It has been realized that with present computing power, coherent all-sky full frequency search in the bandwidth (BW) of the detector of a month long data is computationally prohibitive. Some computationally efficient alternative approaches have been suggested viz. *tracking* and *stacking* (Brady and Creighton 2000; Schutz 1998). *Tracking* involves the tracking of lines in the time-frequency plane built from the Fourier transform (FT) of one day long stretches of data while *stacking* involves dividing the data into day long stretches, searching each stretch for signals, and enhancing the detectability by incoherently summing the FT of data stretches. Also, accurate modeling of GW form, optimal data processing and efficient programming are an integral part of all sky-search.

The basic method to analyze the detector output to get the signature of GW signals rely
on how efficiently one can Fourier analyze the data. Fourier analysis of the data has the advantage of incorporating the interferometers noise spectral density. The problem for the search of CGW largely depend on how accurately one can take into account the translatory motion of the detector acquired from the motions of the Earth in Solar system barycentre (SSB) frame. It has been shown (Srivastava and Sahay 2002a,b) that amplitude modulation will only redistribute the power of the frequency modulated (FM) signal in five frequency bands \( f \pm 2f_{\text{rot}}, f, f \pm f_{\text{rot}} \), where \( f \) and \( f_{\text{rot}} \) are the frequencies of the FM signal and the rotational frequency of the Earth respectively. Hence it is sufficient to consider only FM signal for the analysis of the matching of signals from different locations in the sky.

The study of the matching of signals from locations in the sky that differ in their co-latitude and longitude by \( \pi \) radians has been made for 1-d data set (Srivastava and Sahay 2002c). They observed symmetries in the sky locations. In this paper we report that the observed symmetries are not generic. We study this problem for longer observation time and the other parameters which affect the symmetries. In the next Section we briefly review the FT of FM signal. In Section 3, using the concept of fitting factor (FF), we investigate the matching of signals in an all sky-search with reference to the Earth based laser interferometric detector by considering the source location as the parameter of signal manifold and templates corresponding to different source locations for longer observation time. We present our conclusions in Section 4.

2 Fourier transform of the frequency modulated continuous gravitational wave

The time dependence of the phase of a monochromatic CGW signal of frequency \( f_o \) observed at detector location is given as (Srivastava and Sahay 2002a)

\[
\Phi(t) = 2\pi f_o \left[ t + \frac{R_{se}}{c} \sin \theta \cos \phi + \frac{R_e}{c} \sin \alpha \{ \sin \theta (\sin \beta \cos \epsilon \sin \phi + \cos \phi \cos \beta) + \sin \beta \sin \epsilon \cos \theta \} - \frac{R_{se}}{c} \sin \theta \cos \phi - \frac{R_e}{c} \sin \alpha \{ \sin \theta (\sin \beta_o \cos \epsilon \sin \phi + \cos \phi \cos \beta_o) + \sin \beta_o \sin \epsilon \cos \theta \} \right]
= 2\pi f_o t + Z \cos(a\xi_{\text{rot}} - \phi) + \mathcal{N} \cos(\xi_{\text{rot}} - \delta) - \mathcal{R} - \mathcal{Q} \quad (1)
\]
where
\[
\begin{align*}
\mathcal{P} &= 2\pi f_0 \frac{R_e}{c} \sin \alpha (\cos \beta_o \sin \epsilon \sin \phi + \cos \epsilon \sin \phi) - \sin \beta_o \sin \theta \cos \phi, \\
\mathcal{Q} &= 2\pi f_0 \frac{R_e}{c} \sin \alpha (\sin \beta_o \sin \epsilon \sin \phi + \cos \epsilon \sin \phi) + \cos \beta_o \sin \theta \cos \phi, \\
\mathcal{N} &= \sqrt{\mathcal{P}^2 + \mathcal{Q}^2}, \\
\mathcal{Z} &= 2\pi f_0 \frac{R_{se}}{c} \sin \theta, \\
\mathcal{R} &= \mathcal{Z} \cos \phi,
\end{align*}
\]
\begin{align*}
\delta &= \tan^{-1} \frac{\mathcal{P}}{\mathcal{Q}}, \\
\phi' &= w_{orb} t - \phi, \\
\beta &= \beta_o + w_{rot} t, \\
\xi_{orb} &= w_{orb} t = a \xi_{rot}; \quad a = w_{orb} / w_{rot} \approx 1/365.26, \\
\xi_{rot} &= w_{rot} t
\end{align*}
\] (2)

The two polarisation states of the signal can be taken as
\[
\begin{align*}
h_+(t) &= h_{\alpha+} \cos[\Phi(t)], \\
h_\times(t) &= h_{\alpha\times} \sin[\Phi(t)]
\end{align*}
\] (3)

where \(R_e, R_{se}, w_{rot} \) and \(w_{orb} \) represent respectively the Earth’s radius, the average distance between the centre of Earth from the origin of SSB frame, the rotational and the orbital angular velocity of the Earth. \(\epsilon \) and \(c \) represent the obliquity of the ecliptic and the velocity of light. \(\alpha \) is the colatitude of the detector. Here \(t \) represents the time in s elapsed from the instant the Sun is at the Vernal Equinox and \(\beta_o \) is the local sidereal time at that instant, expressed in radians. \(\theta \) and \(\phi \) denote the celestial colatitude and celestial longitude of the source. These coordinates are related to the right ascension, \(\bar{\alpha} \) and the declination, \(\bar{\delta} \) of the source via
\[
\begin{align*}
\cos \theta &= \sin \bar{\delta} \cos \epsilon - \cos \bar{\delta} \sin \epsilon \sin \bar{\alpha} \\
\sin \theta \cos \phi &= \cos \bar{\delta} \cos \bar{\alpha} \\
\sin \theta \sin \phi &= \sin \bar{\delta} \sin \epsilon + \cos \bar{\delta} \cos \epsilon \sin \bar{\alpha}
\end{align*}
\] (4)

The two polarisation states of the signal can be taken as
\[
\begin{align*}
h_+(t) &= h_{\alpha+} \cos[\Phi(t)] \\
h_\times(t) &= h_{\alpha\times} \sin[\Phi(t)]
\end{align*}
\] (5)
$h_{o+}$, $h_{o\times}$ are the time independent amplitude of $h_+(t)$, and $h_\times(t)$ respectively.

For the analysis of the matching of signals from different locations in the sky, it is sufficient to consider either of the polarisation states of FM signal. We consider the + polarisation of the signal of unit amplitude. Therefore, the FT for a data of $T_{\text{obs}}$ observation time may be expressed as (Srivastava and Sahay 2002b)

$$\tilde{h}(f) = \int_0^{T_{\text{obs}}} \cos[\Phi(t)] e^{-i2\pi ft} dt$$

$$\simeq \frac{\nu}{2 w_{\text{rot}}} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{iA} B[\tilde{C} - i\tilde{D}] ;$$

(7)

where

$$\nu = \frac{f_0-f}{f_{\text{rot}}}$$

$$A = \frac{(k+m)\pi}{2} - \mathcal{R} - \mathcal{Q}$$

$$B = \frac{J_{k+1}(Z)J_{m}(N)}{\nu^2-(ak+m)^2}$$

$$\tilde{C} = \sin \nu\xi_o \cos(ak\xi_o + m\xi_o - k\phi - m\delta)$$

$$-\frac{ak+m}{\nu} \{ \cos \nu\xi_o \sin (ak\xi_o + m\xi_o - k\phi - m\delta) + \sin (k\phi + m\delta) \}$$

$$\tilde{D} = \cos \nu\xi_o \cos (ak\xi_o + m\xi_o - k\phi - m\delta)$$

$$+ \frac{ka+m}{\nu} \sin \nu\xi_o \sin (ak\xi_o + m\xi_o - k\phi - m\delta) - \cos (k\phi + m\delta)$$

$$\xi_o = w_{\text{rot}}T_{\text{obs}}$$

(8)

J stands for the Bessel function of first kind. The computational strain to compute $\tilde{h}(f)$ from equation (7) can be reduced by $\approx 50\%$ by using the symmetrical property of the Bessel functions, given as

$$\tilde{h}(f) \simeq \frac{\nu}{w_{\text{rot}}} \left[ \frac{J_o(Z)J_o(N)}{2\nu^2} \left\{ \sin(\mathcal{R} + \mathcal{Q}) - \sin(\mathcal{R} + \mathcal{Q} - \nu\xi_o) \right\} + \right.$$
\[ \begin{align*}
\mathbf{X} &= \sin(\mathbf{R} + \mathbf{Q} - m\pi/2) \\
\mathbf{Y} &= \cos(\mathbf{R} + \mathbf{Q} - m\pi/2) \\
\mathbf{U} &= \sin \nu \xi_o \cos m(\xi_o - \delta) - \frac{m}{\nu} \{\cos \nu \xi_o \sin m(\xi_o - \delta) - \sin m\delta\} \\
\mathbf{V} &= \cos \nu \xi_o \cos m(\xi_o - \delta) + \frac{m}{\nu} \sin \nu \xi_o \sin m(\xi_o - \delta) - \cos m\delta
\end{align*} \] (10)

Equation (9) contains double infinite series of Bessel function. However, we know that the value of Bessel function decreases rapidly as its order exceeds the argument. From equations (2), it can be shown that the order of Bessel function required to compute \( \tilde{h}(f) \) in the infinite series are
\[ k \approx 3133.22 \times 10^3 \sin \theta \left( \frac{f_o}{\text{1kHz}} \right), \] (11)

\[ m \approx 134 \left( \frac{f_o}{\text{1kHz}} \right), \] (12)

3 Matching of the continuous gravitational wave

Matched filtering is the most suitable technique for the detection of signals whose waveform is known. The waveforms are used to construct a bank of templates, which represent the expected signal waveform with all possible ranges of its parameters (source location, ellipticity, etc.). In an all sky search by match filtering, Srivastava and Sahay 2002c observed symmetries under the following transformations
\[ \theta_T \rightarrow \pi - \theta_T \quad 0 \leq \theta_T \leq \pi, \] (13)

\[ \phi_T \rightarrow \pi - \phi_T \quad 0 \leq \phi_T \leq \pi, \] (14)

\[ \phi_T \rightarrow 3\pi - \phi_T \quad \pi \leq \phi_T \leq 2\pi. \] (15)
The above observations are made by studying few cases for 1-d data set. Hence, it will be important to understand and check the generic nature of the matching of signals under the above transformations. To study this problem we use the formula for FF (Apostolatos 1995) which quantitatively describes the closeness of two signals, given as

$$\text{FF} = \frac{\langle h(f)|h_T(f; \theta_T, \phi_T) \rangle}{\sqrt{\langle h_T(f; \theta_T, \phi_T)|h_T(f; \theta_T, \phi_T) \rangle \langle h(f)|h(f) \rangle}}$$

(16)

where $h(f)$ and $h_T(f; \theta_T, \phi_T)$ represent respectively the FTs of the actual signal waveform and the templates. The inner product of two waveform $h_1$ and $h_2$ is defined as

$$\langle h_1|h_2 \rangle = 2 \int_0^\infty \frac{\hat{h}_1^*(f)\hat{h}_2(f) + \hat{h}_1(f)\hat{h}_2^*(f)}{S_n(f)} \, df$$

$$= 4 \int_0^\infty \frac{\hat{h}_1^*(f)\hat{h}_2(f)}{S_n(f)} \, df$$

(17)

where * denotes complex conjugation, denotes the FT of the quantity underneath ($\tilde{a}(f) = \int_\infty^\infty a(t) \exp(-2\pi if t) \, dt$) and $S_n(f)$ is the spectral density of the detector noise. In our analysis we assume the noise to be stationary and Gaussian. It is remarked that to compute the inner product of the signals one would require the BW of the Doppler modulated signal. From the Doppler shift (Srivastava and Sahay 2002a), the BW of Doppler modulated signal may be given as

$$\text{BW} \approx (1.99115 \times 10^{-4} \sin \theta + 3.09672 \times 10^{-6}) f_o.$$  

(18)

### 3.1 Celestial colatitude

Let us consider that GEO600 detector (the position and orientation of the detectors can be found in Jaranowski et al. 1998) receive a CGW signal of frequency $f_o = 0.1$ Hz (unreasonably low frequency has been chosen for illustrative purposes limited by available computational power) from a source located at $(\theta, \phi) = (25^\circ, 20^\circ)$. In order to evaluate the matching of the signals in colatitude, we first maximize FF over $\phi$ by choosing $\phi = \phi_T = 20^\circ$. Now, we wish to check the symmetries in colatitude represented by equation (13) for the data set $T_{obs} = 120$ d. For the purpose we maximize the FF over $\theta$ by varying $\theta_T$ in discrete steps over entire range i.e. $0^\circ$ to $180^\circ$. For the present case it is sufficient to take the ranges of $k$ and $m$ as 1 to 345 and $-3$ to $3$ respectively and $\text{BW} = 20.1954 \times 10^{-6}\text{Hz}$. 7
The results so obtained are shown in Figure (1). To establish the observed symmetries, we similarly compute the FF for $T_{\text{obs}} = 1, 2, 3, \ldots, 365$ d and observe that the matching of signals remains almost same. However, due to obliquity of the ecliptic, the variation in the matching of signals will depend on source frequency, colatitude and detector position and orientation. We check the dependence of the FF on this parameters. The result so obtained for different Earth based laser interferometric detectors are shown in the Tables (1), & (2) and Figures (2) & (3).

The analysis of the matching of signals in $\theta$ space shows that

(i) for fixed $f_o$, the FF

(a) is independent of $T_{\text{obs}}$ and $\phi$.

(b) does not vary significantly with the variation of source location, detector position and orientation [Tables (1), and (2)].

(ii) the FF falls with the source frequency [Figures (2) and (3)]. From the figure we find that it may not be relevant to take care of the symmetries in the sky locations for the search of CGW from the output of LIGO-I, GEO600 and TAMA detectors whose lower cut off frequency is 40/75 Hz (Owen and Sathyaprakash 1999).

(iii) the approximate fall of FF based on the figures (2) and (3) may be given as

$$\text{FF} = A_o + A_1 \left( \frac{f_o}{\text{Hz}} \right) - A_2 \left( \frac{f_o}{\text{Hz}} \right)^2 + A_3 \left( \frac{f_o}{\text{Hz}} \right)^3$$

where $A_o, A_1, A_2, A_3$ are constants given in Table (2).

### 3.2 Celestial longitude

The Doppler shift due to the motions of the Earth is mainly depend on the colatitude and source frequency and have very less dependence on the longitude. Consequently, grid spacing of the templates for matched filtering for an all sky search will insignificantly depend on longitude (Brady & Creighton 2000). Keeping this in view we similarly check the matching of signals under the transformation given by the equation (14). We chosen the LIGO detector located at Livingston, selected a data set $T_{\text{obs}} = 120$ d, $(\theta, \phi) = (0.5^\circ, 40^\circ)$,
Figure 1: Variation of FF with $\theta_T$.

Figure 2: Fall of FF with $f_o$. 

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Figure 3: Fall of FF with $f_o$.

Table 1: Matching of the signals of frequency 1 Hz under the transformation represented by equation (13) for GEO600 detector.

<table>
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<tr>
<th>$\theta_o$</th>
<th>$\theta_T^o$</th>
<th>FF ($\beta_o = 0^\circ$)</th>
<th>FF ($\beta_o = 90^\circ$)</th>
<th>$\theta_o$</th>
<th>$\theta_T^o$</th>
<th>FF ($\beta_o = 0^\circ$)</th>
<th>FF ($\beta_o = 90^\circ$)</th>
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\begin{table}
\begin{tabular}{|c|c|c|c|c|}
\hline
Detector & $A_0 \times 10^{-3}$ & $A_1 \times 10^{-5}$ & $A_2 \times 10^{-5}$ & $A_3 \times 10^{-7}$ \\
\hline
GEO600($\beta_o=0^o$) & 1000.02 & 124.524 & 129.030 & 137.774 \\
GEO600($\beta_o=90^o$) & 997.796 & 175.426 & 248.072 & 282.478 \\
LIGO Hanford($\beta_o=0^o$) & 998.450 & 225.359 & 172.731 & 221.187 \\
LIGO Hanford($\beta_o=90^o$) & 996.668 & 266.811 & 324.073 & 440.729 \\
VIRGO($\beta_o=0^o$) & 998.548 & 243.480 & 191.503 & 259.026 \\
VIRGO($\beta_o=90^o$) & 996.064 & 316.288 & 362.630 & 529.531 \\
TAMMA300($\beta_o=0^o$) & 997.914 & 342.746 & 249.527 & 390.874 \\
TAMMA300($\beta_o=90^o$) & 995.744 & 371.193 & 458.856 & 756.536 \\
LIGO Livingston($\beta_o=0^o$) & 998.815 & 396.783 & 285.368 & 478.932 \\
LIGO Livingston($\beta_o=90^o$) & 995.622 & 399.087 & 516.416 & 904.211 \\
\hline
\end{tabular}
\caption{Coefficients of the fall of FF with $f_o$ under the transformation represented by equation (13) for $\beta_o = 0^o$ and $90^o$.}
\end{table}

$f_o = 5$ Hz. In order to compute the FF, we first maximize equation (16) over $\theta$ by selecting $\theta = \theta_T = 0.5^o$, followed by maximization over $\phi$ in discrete steps over its entire range, $0^o \leq \phi \leq 360^o$. The result so obtained is shown in Figure (4). We also check the mismatch of the signals for different $\theta$, $\phi$ and $f_o$ by computing the FF for the data set of $T_{obs} = 1, 2, ..., 25/100$ d. The results so obtained are shown in Figures (5), (6), and (7) respectively. Almost same behavior has been observe for the transformation represented by the equation (15).

From the figures we note that the matching of the signals in longitude decreases with $T_{obs}$, $f_o$, $\theta$ and $\phi$. However, behavior of the matching of signals are similar in nature and may be represented by

$$FF = B_o - B_1T_{obs} + B_2T_{obs}^2 - B_3T_{obs}^3 + B_4T_{obs}^4; \quad T_{obs} = 1, 2, ..., 25/100 \text{ d} \quad (20)$$

Where, $B_o$, $B_1$, $B_2$, $B_3$, $B_4$ are the constants given in Table (3). The equations (19) and (20) does not represents the oscillatory part of the figures. This oscillatory behavior is more or less typical in waveform that match well or bad depending on their parameters and is low enough in comparison to the threshold of the detection of GW that one may choose.
<table>
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<th>$\theta_1^o$</th>
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Table 3: Coefficients of the fall of FF for LIGO Livingston detector under the transformation represented by equations (14) and (15) for different $\theta$ and $f_o$. 
Figure 4: Variation of FF with $\phi_T$.

Figure 5: Fall of FF with $T_{obs}$ for different $f_o$.

Figure 6: Fall of FF with $T_{obs}$ for different $\theta$.

Figure 7: Fall of FF with $T_{obs}$ for different $\phi$ and $\phi_T$. 
4 Conclusion

In view of blind all sky search for CGW, we have studied the matching of signals from different source locations assuming the noise to be stationary and Gaussian. For fixed $f_o$, we observed that the matching of signals from locations in the sky that differ in their colatitude by $\pi$ radians is independent of $T_{obs}$, and $\phi$. However, it falls with $f_o$. But in longitude the matching of signals falls with $T_{obs}$, $\theta$, $\phi$, $f_o$. We believe that the matching of signals will increase for the real data. This is due to the resolution provided by the Fast Fourier transform (FFT). However, it may not be relevant to account this symmetries for the search of CGW from the output of LIGO-I, GEO600 and TAMA detectors.

This analysis will be more relevant if one performs hierarchical search (Mohanty & Dhurandhar 1996; Mohanty 1998). This search is basically a two step search, in first step the detection threshold is kept low and in the second step a higher threshold is used. The higher threshold is used for those templates which exceed the first step threshold. The study of the matching of signals in different source locations will be also relevant for Laser interferometer space antenna (LISA) (Hough J., 1995). The work has been initiated and may be useful for the data analysis of CGW.

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References


