The pattern speed of the nuclear disk of M31 using a variant of the Tremaine–Weinberg method

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ABSTRACT

The twin peaks in the nucleus of M31 have been interpreted by Tremaine as a thick, eccentric, disk of stars orbiting a massive dark object; the required alignment of the apoapsides of the stellar orbits could be maintained by self-gravity, and the whole structure might be a discrete, nonlinear eigenmode. The pattern speed of this mode could, in principle, be determined by the Tremaine–Weinberg (TW) method, which requires measurements of the surface brightness, and radial velocity along a strip parallel to the line of nodes. However, spectroscopic observations along the line of nodes are not available. We propose a variant of the TW method, which exploits a basic feature of the eccentric disk model, to extract estimates of the pattern speed from Hubble Space Telescope spectroscopic data, taken along the line joining the two peaks. Within limitations imposed by the data, we estimate that the pattern rotates in a prograde manner and, for an assumed disk inclination of $77^\circ$, the pattern speed $|\Omega_p| < 30 \text{ km s}^{-1} \text{ pc}^{-1}$, or period more than 200,000 years.

Subject headings: galaxies: individual (M31)—galaxies: kinematics and dynamics—galaxies: nuclei

1. Introduction

The nucleus of M31 was first resolved by the Stratoscope II balloon–borne telescope (Light, Danielson, and Schwarzschild 1974), which showed that the peak brightness was displaced relative to the center, as inferred from the isophotes of the outer parts of the galaxy. This was confirmed, and extended by Hubble Space Telescope (HST) observations, which revealed two peaks in the brightness, separated by $0''.49$ (Lauer et al. 1993, King,
Ground–based, as well as HST spectroscopy, have probed the structure of the radial velocities and velocity dispersions, in increasing detail, along many strips across the nuclear region (Dressler and Richstone 1988, Kormendy 1988, Bacon et al. 1994, van der Marel et al. 1994, Gerssen, Kuijken, and Merrifield 1995, Statler et al. 1999, Kormendy and Bender 1999). These provide evidence for the presence of a massive dark object (MDO), which could be a supermassive black hole, of mass $\sim 3 \times 10^7 M_\odot$, located very close to the fainter peak (P2). The dynamical center of the nucleus is believed to coincide with the center of the isophotes of the bulge of M31; this point has been estimated to lie between the two peaks.

Tremaine (1995) proposed that the nucleus could be a thick eccentric disk, composed of stars on nearly Keplerian orbits around the MDO, with their apoapsides aligned in the direction toward the brighter peak (P1); the brightness of P1 is then explained as the increased concentration of stars, resulting from their slow speeds near their apoapsides. He also showed that this model is broadly consistent with the kinematics, as inferred from the spectroscopic observations of Kormendy (1988), and Bacon et al. (1994). Recent work has not only produced further support for Tremaine’s model (Statler et al. 1999, Kormendy and Bender 1999), but has stimulated variations on the basic theme (Statler 1999). Tremaine also suggested that the alignment could be maintained by the self–gravity of the disk, wherein the eccentric distortion could arise as a discrete, nonlinear eigenmode, with some nonzero pattern speed ($\Omega_p$), equal to the common apsidal precession rate. The dynamical question is yet to be resolved in a self–consistent manner, although explorations of orbits in model potentials have identified a family of resonant, aligned loop orbits, which could serve as building blocks (Sridhar and Touma 1999, Statler 1999); reasonably faithful reproduction of the nuclear rotation curve adds some degree of confidence (Statler 1999). If the nuclear disk is indeed a steadily rotating, nonlinear eigenmode, what is $\Omega_p$?
Tremaine and Weinberg (1984, hereafter TW) invented a method of estimating the pattern speed of a barred disk galaxy, that uses measurements of the surface brightness, and radial velocity along a strip parallel to the line of nodes (defined as the line of intersection of the disk and sky planes). That this methods works was proved when the pattern speed of the bar in NGC 936 was estimated by Merrifield and Kuijken (1995). Unfortunately, the radial velocity measurements of the nucleus of M31 (see references above on spectroscopy) are available on strips that, either do not coincide with the line of nodes, or possess too poor an angular resolution for direct application of the TW method. In this Letter, we show that the HST observations of Statler et al. (1999, hereafter SKCJ), together with Tremaine’s (1995) model, can be used to estimate $\Omega_p$.

2. A modified Tremaine–Weinberg method applicable to M31

We briefly recall TW’s derivation of their kinematic method. Let us assume that the disk is razor–thin, with a well–defined pattern speed, $\Omega_p$. The plane of the disk is assumed to be inclined at angle $i$ with respect to the plane of the sky. Let $(x, y)$, and $(r, \phi)$ be cartesian and polar coordinates, respectively, in the plane of the disk, with the origin coinciding with the center of mass of the system (disk plus MDO). Let the cartesian coordinates in the sky plane be $(X, Y) = (x, y \cos i)$; the $x$ and $X$ axis coincide with the line of nodes. The disk is assumed to rotate steadily, hence the surface brightness of stars, $\Sigma(x, y, t) = \tilde{\Sigma}(r, \phi - \Omega_p t)$. The surface brightness is also assumed to obey a continuity equation, without a source term. Let $v$ be the (mean) velocity field in the disk plane. The continuity equation can be manipulated to yield,

$$\nabla \cdot [(v - u) \Sigma] = 0,$$

(1)

where $u = \Omega_p (-y \hat{x} + x \hat{y})$. TW proceed by integrating equation (1) over $x$ from $-\infty$ to $+\infty$. Assuming $\Sigma \to 0$ sufficiently rapidly as $|x| \to \infty$, and integrating over $y$ from $y$ to
\( \int_{-\infty}^{\infty} dx \Sigma \) extends to \( \int_{-\infty}^{\infty} dx \Sigma v_y \). Noting that the sky brightness, \( \Sigma_s = \Sigma / \cos i \), and the radial velocity, \( V_\parallel = v_y \sin i \), the integrals may be expressed in terms of observable quantities; hence \( \Omega_p \) can be estimated when \( V_\parallel \) has been measured on a strip parallel to the line of nodes.

SKCJ measured \( V_\parallel \) along the P1–P2 line, which is inclined by about \( 4^\circ \), in the sky plane, to the line of nodes. Therefore \( V_\parallel \) is available on a strip that makes an angle, \( \theta \simeq 4^\circ / \cos 77^\circ \simeq 18^\circ \), with the \( x \)-axis, in the disk plane; the TW procedure needs some modification, before \( \Omega_p \) can be extracted. Let \( x' = x \cos \theta + y \sin \theta \) and \( y' = -x \sin \theta + y \cos \theta \) be the rotated cartesian coordinates. Let us also denote the surface brightness by \( \Sigma'(x', y', t) \). The SKCJ measurements are along the strip \( y' = 0 \), that passes through the origin. Equation (1) expresses a relation that is invariant under rotation of cartesian coordinates. Hence application of the TW procedure provides an identical relationship between the integrals over the strip, defined by \( y' = 0 \):

\[
\Omega_p \int_{-\infty}^{\infty} dx' \Sigma' x' = \int_{-\infty}^{\infty} dx' \Sigma' v_y'
\]

We now express the integrals in terms of observable quantities: \( x' = X \cos \theta + Y \sin \theta / \cos i \) and \( 0 = y' = -X \sin \theta + Y \cos \theta / \cos i \) can be used to eliminate \( Y \), giving \( x' = X / \cos \theta \). As before, \( \Sigma'(x', y', t) = \cos i \Sigma_s(X, Y, t) \). However, the radial velocity,

\[
V_\parallel = \sin i \left( v_y' \cos \theta + v_x' \sin \theta \right),
\]

depends on \( v_y' \) as well as \( v_x' \). Hence it is, in general, not possible to express \( v_y' \) in equation (2) in terms of \( V_\parallel \) alone (the TW method finesses this problem because \( \theta = 0 \) kills the contribution to \( V_\parallel \) from the \( v_x' \) term). However, we are able to make progress by recalling the basic elements of Tremaine’s (1995) model. If the nuclear disk is largely composed of nearly Keplerian ellipses, with their apoapsides aligned along the P1–P2 line, and we choose \( \theta \) such that our strip lies along this line, then the strip intersects all these ellipses at right
angles. Thus $v'_x = 0$, and we recover the familiar TW relation,

$$\Omega_p \sin i \int_{-\infty}^{\infty} d\Sigma_s X = \int_{-\infty}^{\infty} d\Sigma_s V_\parallel. \quad (4)$$

It should be noted that the value of $\theta$ drops out of equation (4). Moreover, $v'_x = 0$ even when the apsides of the ellipses precess. Below we use HST photometry and kinematics to estimate $\Omega_p$.

3. $\Omega_p$ from HST photometry and spectroscopy

HST photometric data, reported in Lauer et al. (1993), was kindly supplied to us by Prof. Ivan King. Figure 1 shows the (sky) surface brightness along the P1–P2 line. The contribution from the bulge of M31 was subtracted using two different fits to the bulge brightness, namely a Nuker fit as described in Tremaine (1995), and a Sérsic fit as described in Kormendy and Bender (1999). The disks so obtained will henceforth be referred to as a “T–disk”, and a “KB–disk”, respectively.

SKCJ observed the stellar kinematics along the P1–P2 line, using the f/48 long-slit spectrograph of the HST Faint Object Camera. We obtained radial velocities along the P1–P2 line, including the errors on them, from their “de–zoomed” rotation curve (given in Figure 3, as well as Table 1 of SKCJ); these are displayed in Figure 2a. The integrals in equation (4) need to be computed, with upper and lower limits symmetrically displaced about the center of mass of the disk plus MDO. Kormendy and Bender (1999) have determined the center of mass to be displaced by $0''.098$ from P2 toward P1, and we use this value in the computations for Figures 2b and 2c. The stated errors on the radial velocities were used by us to generate 300 random realizations. For each of these realizations of the rotation curve, we evaluate the integrals for five different limits, ranging from $\pm 0''.8$ to $\pm 1''.2$. As is clear from equation (4), only the combination, $\Omega_p \sin i$ can be determined.
It is a common assumption that the nuclear disk of M31 has the same inclination, to the sky plane, as the much larger galactic disk of M31, which is inclined at $\simeq 77^\circ$. We wish to state our results independent of this assumption, so we present estimates of the quantity, $\tilde{\Omega}_p = (\sin i / \sin 77^\circ) \Omega_p$, in Figures 2b–2d.

Figures 2b and 2c display the estimates of $\tilde{\Omega}_p$ so obtained, together with $1\sigma$ error bars, as a function of the limits of the integrals, for the T–disk and KB–disk, respectively. It is evident that the estimates of pattern speed do not vary much when the limits of integration lie between $\pm 1''$ and $\pm 1''.2$. We also explore the dependence on the position of the center of mass of the system, assumed in the computation of the integrals. Although we used the most recent determination, due to Kormendy and Bender (1999), it must be noted that earlier work (Lauer et. al. 1993, King, Stanford, and Crane 1995, Tremaine 1995) records smaller values, $\leq 0''.05$, away from P2 toward P1. Figure 2d plots our estimates of $\tilde{\Omega}_p$, for a range of values of the center of mass, with limits of integration fixed at $\pm 1''$.0. A negative value of $\tilde{\Omega}_p$ means that the pattern is *prograde*; Figures 2b–2d indicate that this is the most likely possibility, with the absolute value increasing with the separation between P2 and the center of mass of the system. This is reasonable, because a larger separation corresponds to a larger disk mass, which implies a greater contribution from the self–gravity of the disk, which is ultimately responsible for the (alignment and) precession of the disk. The errors on $\tilde{\Omega}_p$ are large, and a non rotating pattern cannot be ruled out with certainty. Below we quote representative bounds on the absolute value of the prograde pattern speed:

$$|\Omega_p| \leq \frac{\sin 77^\circ}{\sin i} \left\{ \begin{array}{ll}
34 \pm 8 \text{ km s}^{-1} \text{ pc}^{-1}, & \text{T–disk;} \\
20 \pm 12 \text{ km s}^{-1} \text{ pc}^{-1}, & \text{KB–disk.}
\end{array} \right.$$  \hspace{1cm} (5)

For each realization of the rotation curve, the zero–crossing point (the “rotation center”) was determined by a third–order spline interpolation. We found the position of the rotation center to be displaced by $0''.17 \pm 0''.01$, from P2 toward P1. This should be compared with the value of $0''.16 \pm 0''.05$, quoted by SKCJ. We also tested for any systematic dependence
of $\Omega_p$ on the position of the rotation center, and found none.

4. Conclusions

Our estimate of the pattern speed of the nuclear disk of M31 should be qualified by a discussion of possible sources of errors, most of which are difficult to estimate quantitatively. SKCJ calibrate velocities relative to an average over an 8 arcsec$^2$ aperture centered on the nucleus (Ho, Filippenko, and Sargent 1993), and assure us that the errors are likely to be small. More significant, perhaps, are the systematic errors in $V_\parallel$, mentioned by SKCJ; these are shown as open squares in Figure 2a. SKCJ used a slit of width 0.063, and this will introduce contributions to $V_\parallel$ from nonzero values of $v'_x$ (see equation (3)). This effect is somewhat mitigated by cancellation between positive and negative values of $v'_x$, and the fact that the width of 0.063 is of much smaller scale than the minor axis, projected onto the sky plane, of the smallest ellipse used by Tremaine (1995; see Figure 2a of his paper) to represent the nuclear disk. The limits of integration are necessarily finite in numerical computation, and we have resisted the temptation to include corrections by extrapolation of the $\Sigma_\ast$ and $V_\parallel$ profiles.

A basic assumption underlying the application of a TW–like method is that the surface brightness obeys a continuity equation, which would be valid for a stellar disk in the absence of star formation (or death). We expect the numbers of stars to be conserved, except possibly in the vicinity of P2, where the observed UV excess has been interpreted as contributions from early–type stars (King, Stanford and Crane 1995, Lauer et al. 1998). However, these stars do not contribute much to the photometric and kinematic data we have used. Tremaine (1995) argues that two–body relaxation is expected to thicken the disk, whereas we assumed that the disk was razor–thin. The original TW method is applicable to thick disks, so long as the streaming velocity normal to the disk plane is zero.
In addition to the assumption of zero normal streaming velocity, let us suppose that the three dimensional density, $\rho$, is symmetric about the midplane of the disk. The contribution to $V_\parallel$ from $v'_x$ arise from an integral along the (inclined) line of sight that runs through the thick disk. Consider two points along this line of sight that are equally displaced about the midplane of the disk. Aligned, nearly Keplerian orbits have flows such that $\rho$ is equal, whereas $v'_x$ is equal and opposite at these two points; in this ideal picture, there is pair-wise perfect cancellation, and no net contribution to $V_\parallel$ from $v'_x$. In practice there should be some cancellation, because $v'_x$ will have opposite signs at two corresponding points, but there could be a net contamination from the unequal values of $|v'_x|$ and $\rho$.

Tremaine’s original model, which was a reasonable fit to the then available photometry and kinematics, considered a non rotating disk, and it would be appropriate to inquire about the implications of a non zero pattern speed. A pattern that is prograde with angular speed, say, of 20 km s$^{-1}$ pc$^{-1}$ would contribute only about 35 km s$^{-1}$ to the radial velocity at P1, which is about 250 km s$^{-1}$, according to the measurements of SKCJ. The maximum radial velocity quoted by Tremaine (1995) is less than 200 km s$^{-1}$, so a non zero pattern speed could still be accommodated. Our estimates do not rule out a non rotating disk, but we would like to offer a physical argument in support of a non zero pattern speed. For the disk plus MDO to be in a steady, non rotating state, the gravitational force on the MDO should necessarily vanish. Our (unpublished) numerical computations indicate that the force is indeed non zero.

A limitation of our method is that it uses, in an essential manner, the assumption that most of the contribution to $V_\parallel$ comes from orbits which intersect the measurement strip at right angles. In comparison, the original TW method does not rely on assumptions about the geometry of the mean flow; averaging over several strips, all parallel to the line of nodes, will improve estimates of the pattern speed, as Merrifield and Kuijken (1995)
demonstrated. Thus it is necessary to verify our estimates of $\Omega_p$, by using the TW method on future observations of $V_\parallel$, together with better photometry such as Lauer et al. (1998), along strips parallel to the line of nodes. An extremely useful set of observations that could be performed would be two-dimensional spectroscopy, similar to the work of Bacon et al. (1994), with the increased angular resolution that should be available in the near future.

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Fig. 1.— V–band surface brightness along the P1–P2 line, with P2 located at $X = 0$, and P1 at $X = -0''.49$. The solid line connects data points taken from Lauer et al. (1993). The dotted and short–dashed curves are the contributions from Nuker (Tremaine 1995) and Sérsic (Kormendy and Bender 1999) bulges, respectively. The corresponding estimates for the surface brightness of the disk (T–disk and KB–disk) are given by the long–dashed, and dot–dashed curves.
Fig. 2.— (a) Radial velocity measurements taken from SKCJ. (b, c) $\tilde{\Omega}_p$ plotted versus the truncation radius of the disk, as measured from the center of mass of the system, for the T–disk and KB–disk, respectively; the center of mass was taken to be $0''.098$ away from P2 toward P1. (d) $\tilde{\Omega}_p$ plotted versus the assumed location of the center of mass of the system, for a disk truncation radius equal to $1''.0$. 
REFERENCES


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