COSMIC MICROWAVE BACKGROUND SPECTRUM AND G-VARYING COSMOLOGY

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It is shown that G-varying cosmologies provide a better fit to the observed data on cosmic microwave background, than the standard Friedmann models.

1. Introduction. Woody and Richards [1] have measured the spectrum of microwave background radiation (MBR) in the frequency range 1.7 to 40 cm$^{-1}$ and have argued that the data show deviations from a simple black body curve. From the point of view of the big bang cosmology it is necessary that the MBR spectrum should show a strict planckian form. Any deviations from this predicted form must either be explained as simple perturbations of (relatively recent) astrophysical origin or dismissed as due to some systematic uncorrected errors in the data.

It is also possible to incorporate a third alternative if we decide to widen the framework of big bang theory beyond the models given by general relativity. In particular if we consider cosmological models with varying gravitational constant $G$, some of these models also interpret MBR as the relic of a big bang origin of the universe. There is the possibility in these models that the predicted MBR spectrum is non-planckian; in which case it is of interest to enquire whether the fit between theory and observation is significantly improved in G-varying cosmologies. We report here the result of such an analysis which leads us to believe that G-varying cosmologies do fit the present data very well.

2. MBR spectrum in G-varying cosmologies. Canuto and Hsieh [2] have made a detailed study of the spectrum of MBR in Dirac cosmologies using different gauges. More recently Canuto and Narlikar [3] have considered the shape of the spectrum in the G-varying Hoyle–Narlikar cosmology [4]. In both these investigations, the spectrum turns out to have the following form in the usual notation:

$$\mathcal{F}(v) = \mu(t)(2\hbar v^3/c^2)(e^{\hbar v/kT} - 1)^{-1}. \quad (1)$$

Here $\mu(t)$ is an epoch dependent factor which multiplies the standard planckian spectrum. In the Dirac cosmologies this factor depends on $G(t)$ and the gauge function $\beta(t)$ discussed by Canuto and Hsieh [2]. In the Hoyle–Narlikar cosmology with matter creation, we get (cf. ref. [3]),

$$\mu(t) = G(t_\infty)/G(t). \quad (2)$$

Here $G(t_\infty)$ is a constant evaluated at some fixed epoch.

The important aspect to note here is that the MBR spectrum depends on two epoch dependent parameters $\mu$ and $T$ instead of the one parameter $T$ which characterizes the planckian spectrum in Friedmann cosmologies.

3. The fit between theory and observations. One point is immediately clear from fig. 2 of Woody and Richards [1], viz. the observed background has a narrower peak than that predicted by the Planck curve at $T = 2.96$ K. It is easily verified that the radius of curvature of the Planck curve at its peak varies as $T^2$. Thus the smaller the temperature the narrower the peak. However, if we lower the value of $T$, the area under the curve decreases and this leads to a disagreement with the overall intensity of radiation. The G-varying cosmologies, on the other hand, supply the additional parameter $\mu$ which takes care of the intensity of radiation if $T$ has to be lowered to match the narrowness of the spectrum. Thus the “best fit” version of eq. (1) is ex-
Table 1  
Summary of analysis. I stands for uncorrelated errors and II for all errors.

<table>
<thead>
<tr>
<th>Curve fitted</th>
<th>Error type</th>
<th>$\mu$</th>
<th>$T$</th>
<th>Likelihood function $L$</th>
<th>$x_0^2$</th>
<th>Confidence limit $P(x^2 &gt; x_0^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard cosmology</td>
<td>I</td>
<td>1</td>
<td>2.93</td>
<td>$5 \times 10^{-22}$</td>
<td>$\approx 81$</td>
<td>$\approx 10^{-13}$</td>
</tr>
<tr>
<td>(planckian)</td>
<td>II</td>
<td>1</td>
<td>2.90</td>
<td>$7 \times 10^{-16}$</td>
<td>$\approx 21$</td>
<td>$\approx 0.11$</td>
</tr>
<tr>
<td>G-varying cosmology</td>
<td>I</td>
<td>1.75</td>
<td>2.54</td>
<td>$3 \times 10^{-6}$</td>
<td>$\approx 8.0$</td>
<td>0.64</td>
</tr>
<tr>
<td>(modified planckian)</td>
<td>II</td>
<td>1.77</td>
<td>2.53</td>
<td>$10^{-11}$</td>
<td>$\approx 2.5$</td>
<td>0.99</td>
</tr>
</tbody>
</table>

expected to give $\mu > 1$ and $T < 2.96 \text{ K}$. These qualitative predictions are borne out by the following analysis.

The data are given in the form of $F$ values at different frequencies. We denote the observed value of $F$ at a frequency $\nu_i$ ($i = 1, \ldots, N$) by $F_i$. Let $\sigma_i$ denote the estimated error of measurement at $\nu_i$. We define the likelihood function by

$$L(T, \mu) = \prod_{i=1}^{N} \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left\{ -\frac{[F_i - \mathcal{F}(\nu_i)]^2}{2\sigma_i^2} \right\},$$

(3)

and we maximize $L$ with respect to $\mu$ and $T$. The same fitting procedure can be adopted for the standard planckian curve which has $\mu = 1$.

Woody and Richards (private communication) quote two types of errors: uncorrelated and all errors. We have applied our analysis to both types, noting that there are 12 data points for which uncorrelated errors are quoted and 16 data points for which all errors are given. Table 1 gives the summary of our analysis. It is clear from the table that the fit with the observations is dramatically improved when we go from the standard to the G-varying cosmologies, whether we consider type I or type II. The last column indicates the probability of obtaining a value of $x^2$ exceeding the observed value $x_0^2$ given by

$$x_0^2 = \sum_{i=1}^{N} \left\{ \frac{[F_i - \mathcal{F}(\nu_i)]}{\sigma_i} \right\}^2.$$

(4)

Thus the relatively high probabilities indicate how good the fit is in the case of G-varying cosmologies. As expected on qualitative grounds, the effective MBR temperature is lowered in this case to $\approx 2.53 \text{ K}$ and the multiplying constant exceeds 1.

The quantity $\mu$ in the Hoyle—Narlikar cosmology measures the ratio of $G$ at a fixed $t_*$ to its value at the present epoch $t_0$. Since $\mu > 1$, and $G$ decreases with epoch we must have $t_* < t_0$. This epoch $t_*$ corresponds to a redshift of $\approx 0.32$.

We draw no other conclusion from this analysis except to point out that G-varying cosmologies do appear to fit the present MBR data better than the standard Friedmann models. It will be interesting to repeat this analysis as and when more improved data with smaller errors are available.

We thank Drs. Woody and Richards for making available to us the Berkeley data which made this analysis possible.

References