It is shown that the theoretical curve of relic radiation in the $G$-varying Hoyle–Narlikar cosmology provides an acceptable fit to the observations at long as well as short wavelengths.

1. Introduction. In an earlier paper [1] we had shown that since the theoretically predicted spectrum of relic radiation in $G$-varying cosmologies has an additional parameter besides temperature, it can give a better fit with the observed cosmic microwave background spectrum than is possible in the standard Friedmann cosmology. In particular, in the Hoyle–Narlikar cosmology [2] the spectral function at cosmic epoch $t$ is given by

$$f(v) = \mu(t) \left(2h/v^3/c^2\right)(e^{h/v/kT} - 1)^{-1},$$

where $\mu(t) = G(t_\ast)/G(t)$, $t_\ast$ being a constant epoch and $G$ the time-dependent gravitational constant.

In ref. [1] we had obtained “the best” values for $T$ and $\mu(t)$ by comparison of (1) with the data by Woody and Richards [3]. This fit was statistically much better (with $\chi^2$ probability as high as 0.99) than the best fit possible by a planckian distribution ($\chi^2$ probability $\leq 0.11$). The maximum-likelihood technique used by us could not, however, be extended to include the data available in the long-wavelength range. Hence our best-fit parameters could not do justice to the Rayleigh–Jeans part of the spectrum. In the present paper we modify our technique to counter this difficulty.

There is added motivation for the present exercise. In a revised assessment of their results by Woody and Richards [4] the departure of observations from the classical Friedmann prediction is claimed to be over $3\sigma$. Further, Smoot et al. [5] have published more observations in the long-wavelength range which overlaps with the range covered by Woody and Richards. We will assess the performance of $G$-varying cosmology of Hoyle and Narlikar vis-a-vis the standard big-bang cosmology in the light of these recent inputs.

2. The fitting technique. The Woody–Richards data gives twelve data points in the form of observed intensity values $F_i$ at frequencies $\nu_i$ and all errors $\sigma_i$. Let $f_i$ denote the theoretical intensity value at $\nu_i$ computed according to (1) for parameters $\mu$ and $T$. The likelihood function to be maximized is defined by

$$L = \prod_{i=1}^{12} \frac{1}{\sqrt{2\pi \sigma_i}} \exp\left[-(F_i - f_i)^2/2\sigma_i^2\right].$$

In ref. [1] we had maximized $L$ with respect to $\mu$ and $T$. However, this procedure ignores the Rayleigh–Jeans part of the spectrum. From (1) we see that the Rayleigh–Jeans part is that corresponding to an effective temperature

$$\bar{T} = \mu T.$$  

Accordingly, we fixed $\bar{T}$ at an ad hoc value and maximized $L$ subject to the constraint (3). Corresponding to the maximum $L$ we then compute $\chi^2$ by the formula

$$\chi^2 = \sum_{i=1}^{12} \left[(F_i - f_i)/\sigma_i\right]^2.$$  

3. Results. Fig. 1 shows how $\chi^2$ varies with $\bar{T}$, the Rayleigh–Jeans temperature. The goodness of fit worsens as $\chi^2$ increases and $\bar{T}$ decreases. To express
the goodness of fit in terms of multiples of probable error $\sigma$ we have given the $\sigma$ multiples (for $\chi^2$ at 11 degrees of freedom) on the upper horizontal axis of fig. 1.

The $1\sigma$ limit is crossed at $T = 3.30 \, \text{K}$ and the $2\sigma$ limit at $T = 2.95 \, \text{K}$. The earlier observations at long wavelengths [6] had longer error bars and the quoted weighted average $2.74 \pm 0.087 \, \text{K}$ does not truly reflect the likelihood of systematic errors being present in some measurements. The $1\sigma$-upper limits of individual measurement in fact range from $2.73 \, \text{K}$ to $4.9 \, \text{K}$. Thus the $1\sigma$ value of the temperature $T$ above does not seem excessively high. However, the recent measurements of Smoot et al. [5] quote $T = 2.79 \pm 0.10 \, \text{K}$, and here the agreement between these observations and the Woody--Richards data as interpreted by us is good to $\sim 2\sigma$ level. Even in this case the $1\sigma$ upper limits quoted by Smoot et al. for the five observations are respectively $2.87 \, \text{K}$, $2.91 \, \text{K}$, $3.08 \, \text{K}$, $3.10 \, \text{K}$ and $3.5 \, \text{K}$.

To conclude, the predicted spectrum in the Hoyle--Narlikar cosmology gives an acceptable fit with the Woody--Richards data at high frequencies as well as with the measurements in the Rayleigh--Jeans part of the microwave background spectrum.

References