\section{1 INTRODUCTION}

Well before the data from the high-redshift supernova project became available, several independent constraints indicated the existence of a cosmological constant (Bagla, Padmanabhan & Narlikar 1996; Ostriker & Steinhardt 1995; Efstathiou, Sutherland & Maddox 1990). In the last decade, observational evidence for an accelerating Universe has become conclusive, with almost all other possibilities being ruled out by observations of high-redshift supernovae (Riess et al. 2004; Barris et al. 2004; Tonry et al. 2003) and the cosmic microwave background radiation (CMBR) (Melchiorri et al. 2000; Spergel et al. 2003). The accelerated expansion of the Universe requires either a cosmological constant or some form of dark energy (Ellis 2003; Padmanabhan 2003; Peebles & Ratra 2003; Sahni & Starobinsky 2000) to drive the acceleration, with $w = p/\rho < -1/3$. Although a cosmological constant is the simplest solution from a phenomenological point of view (requiring just one fine-tuned parameter), there is no natural explanation of the small observed value. This has led theorists to develop models in which a field, typically a scalar field, provides the source of dark energy, e.g. quintessence (de la Macorra & Piccinelli 2000; Gonzalez-Diaz 2002; Urena-Lopez & Matos 2000; de Ritis & Marino 2001; Bludman & Roos 2002; Rubano & Scudellaro 2002; Sen & Seshadri 2003; Steinhardt 2003), $k$-essence (Armendariz-Picon, Mukhanov & Steinhardt 2001; Chiba 2002; Malquarti et al. 2003; Chimento & Feinstein 2004; Scherrer 2004), tachyons (Choudhury et al. 2002; Frolov, Kofman & Starobinsky 2002; Gibbons 2002; Padmanabhan 2002; Shiu & Wasserman 2002; Bagla, Jassal & Padmanabhan 2003; Gibbons 2003a; Jassal 2003; Sen 2003; Sen 2003; Aguirregabiria & Lazkoz 2004; Gorini et al. 2004), phantom fields (Caldwell 2002; Carroll et al. 2003; Dabrowski et al. 2003; Frampton 2003; Gibbons 2003b; Gonzalez-Diaz 2003b; Hao & Li 2003a,b; Nojiri & Odinstov 2003; Singh et al. 2003; Cline et al. 2004), branes (Gonzalez-Diaz 2000; Uzawa & Soda 2001; Burgess 2003; Jassal 2003; Milton 2003), etc. In the absence of significant spatial variation in the dark energy, the key difference between such models and the one with the cosmological constant is that, in general, $w$ is a function of redshift $z$ in the former. Perturbations in dark energy also lead to observable signatures, though these can easily be confused with other physical effects (Caldwell, Dave & Steinhardt 1998; Weller & Lewis 2003; Bean & Doré 2004). There have also been a few proposals for unified dark matter and dark energy (Padmanabhan & Choudhury 2002; Gonzalez-Diaz 2003a; Cardone et al. 2004) but these models are yet to be developed in sufficient detail to allow direct comparison with observations in a fruitful manner. If the current observations had excluded $w = -1$, then one could have immediately ruled out cosmological constant as a candidate; but as this is not the case, direct exploration of $w(z)$ at different redshifts, in order to check possible dependence of $w$ on $z$, is of importance. Observations constrain the entire suite of parameters that describe cosmological parameters and while it is possible to choose other cosmological parameters so that $\Lambda$CDM is not allowed, these models have not been ruled out so far.

It has been known for some time that supernova observations and constraints from structure formation can be combined to put stringent limits on models for dark energy (Perlmutter, Turner & White 1999). Several attempts have been made in the past to constrain the equation of state for dark energy, along with other cosmological parameters, using the observations of galaxy clustering, temperature anisotropies in the CMBR and the high-redshift supernovae (Perlmutter et al. 1999; Lee & Ng 2003; Linder & Jenkins 2003; Pogosyan et al. 2003; Tegmark et al. 2003; Huterer & Cooray 2004; Lee, Lee & Ng 2004; Maccio et al. 2004; Multamaki et al. 2004; Wang & Tegmark 2004).

This work, which is in the same spirit, focuses exclusively on constraining the variation of the equation of state for dark energy by using a combination of such observations. We wish to constrain the variation of dark energy while keeping most of the other...
cosmological parameters fixed around their favoured values. In particular, we study the effect of a varying equation of state on the angular power spectrum of the CMBR fluctuations. Dark energy is not expected to be dynamically significant at the time of decoupling and – in fact – models in which this is not true are plagued by slow growth of density perturbations (Benabed & Bernardeau 2001; Amendola 2003; DeDeo et al. 2003). Nevertheless, evolving dark energy will affect the features of temperature anisotropies in the CMBR in at least two ways: (i) the angular scale of features in temperature anisotropy, such as the peaks, will change because the angular diameter distance depends on the form of \( w(z) \); and (ii) the integrated Sachs–Wolfe (ISW) effect will also depend on the nature of dark energy and its evolution (this effect is more relevant at small \( l \)). Thus observations of temperature anisotropies in the CMBR can be used to constrain the evolution of dark energy. Combined with the supernova observations, this allows us to put tight constraints on the equation of state of dark energy and its evolution. Our approach here is to use the full WMAP angular power spectrum in order to ensure that both the effects mentioned above are captured in the analysis. For lower multipoles \( l \ll 20 \), signals from the ISW effect, perturbations in dark energy and reionisation need to be disentangled, however the relative importance of this part of the angular power spectrum is limited as we use the full angular power spectrum from WMAP. Our aim is to demonstrate that the combination of WMAP observations and high-redshift supernova observations is a very powerful constraint on variations in dark energy, certainly more powerful that either of the observations used in isolation. As far as we know, most previous attempts to constrain the dark energy sector using WMAP data have not used the full angular power spectrum.

2 VARYING DARK ENERGY

Supernova observations, as well as the angular power spectrum of temperature anisotropies in the microwave background put geometrical constraints on dark energy. Thus these observations constrain the Hubble parameter \( H(z) \), which for the models under consideration here can be written as:

\[
H^2(z) = H_0^2 \left[ \Omega_{de}(1 + z)^3 + \Omega_{DE}^{\text{iso}}(z)/\Omega_{0}^{\text{DE}} \right],
\]

where we have ignored the contribution of radiation to energy density. Observed fluxes from supernovae and location of peaks in the CMB constrain distances to supernovae and to the surface of last scattering (Eisenstein & White 2004), respectively. Dark energy can be constrained only at redshifts where it contributes significantly to the energy density, therefore observations are sensitive to changes in \( \Omega_{DE}^{\text{iso}}(z) \) at low redshifts, and variations at high redshifts \( (z \gg 1) \) will be difficult to detect. Variation in \( \Omega_{DE}^{\text{iso}}(z) \) can be written in terms of changes in the equation of state parameter \( w \). Once we allow for variation of \( w \), one is dealing with a function which, technically, has an infinite number of parameters. Given a finite number of observations, one can always fine-tune such a function. From a practical point of view, it is necessary (and often sufficient) to represent \( w(z) \) using a small number of parameters and do the analysis. The results will necessarily have some amount of parametrization dependence (Bassett, Corasaniti & Kunz 2004) but this can be controlled by choosing different forms of parametrization and determining the range of variations in the results. In this work, we shall use the two parametrizations:

\[
w(z) = w_0 + w_1 \frac{z}{(1 + z)^p}; \quad p = 1, 2
\]

For both \( p = 1, 2 \) we have \( w(0) = w_0; \ w'(0) = w_1 \) but the high-redshift behaviour of these functions are different: \( w(\infty) = w_0 + w_1 \) for \( p = 1 \) while \( w(\infty) = w_0 \) for \( p = 2 \). Hence \( p = 2 \) can model a dark energy component which has the same equation of state at the present epoch and at high redshifts, with rapid variation at low \( z \). Observations are not very sensitive to variations in \( w(z) \) for \( z \gg 1 \), hence return to the present value is not of critical importance. However, it does allow us to probe rapid variations at small redshifts. For \( p = 1 \), we can trust the results only if \( w_0 + w_1 \) is well below zero at the time of decoupling so that dark energy is not relevant for the physics of recombination of the evolution of perturbations up to that epoch. As we shall see, the allowed range of parameters does not contain such models. This constraint on the asymptotic value means that this parametrization cannot be used to probe rapid variations at small redshifts.

We will take \( \Omega_{de} + \Omega_{ae} = 1 \) (Melchiorri et al. 2000; Spergel et al. 2003) where density parameter for the total non-relativistic component is denoted by \( \Omega_{de} \) and the density parameter for dark energy is \( \Omega_{ae} \). We will make assumptions regarding a few other parameters as well: (i) we assume that \( \Omega_{de} = 0.05 \); (ii) we assume that \( h = 0.7 \); and (iii) we assume that the density fluctuations have the Harrison–Zel’dovich spectrum, i.e. \( n = 1 \). We do this because (i) there have been numerous studies in the past, restricting the standard cosmological parameters and (ii) we wish to focus on the effect of \( w \neq 0 \). One can quarrel with these assumptions but they are sufficient to give us an idea of the parameter space available for variation of dark energy. A complete study of these issues will need to address two questions: what is the parameter space available for variation of dark energy if all parameters of interest in cosmological models are allowed to vary (this essentially addresses the issue of potential degeneracies of variation of dark energy with other parameters), and if dark energy is allowed to vary then does that increase or change the allowed range of other parameters? We are carrying out a detailed study to address these issues (Jassal, Bagla & Padmanabhan, in preparation). We will comment on dependence of our results on some of these parameters whenever relevant.

For the supernova analysis, which is by now standard (Amendola & Quercellini 2003; Jimenez 2003; Jimenez et al. 2003; Padmanabhan & Choudhury 2003; Alam, Sahni & Starobinsky 2004a; Alam et al. 2004b; Bassett 2004; Bassett et al. 2004; Choudhury & Padmanabhan 2004; Corasaniti et al. 2004; Daly & Djorgovski 2004; Gong 2004; Jonsson et al. 2004; Wang 2004; Wang et al. 2004), we combine the data on 230 supernovae published by Tonry et al. (2003) and also for the 23 supernovae listed by Barris et al. (Tonry et al. 2003; Barris et al. 2004; Riess et al. 2004). To account for the uncertainty in measurement of cosmological redshift, we neglect the data points at redshift \( z < 0.1 \). The points with \( A_P > 0.5 \) are neglected because of the uncertainty in host galaxy extinction, after these cuts we are left with data for 194 supernovae. We compare the distance modulus predicted by theoretical models to the observed variation of apparent magnitudes of high-redshift supernovae. We use the \( \chi^2 \) minimization method to find the best-fitting model. The supernova data favours dark energy to have an equation of state \( -1.3 \leq w(0) \leq -0.7 \) at the present epoch, with the lower values favoured for larger \( \Omega_{ae} \). Significant variation in energy density of dark energy is allowed by supernova observations, e.g. the energy density can change by more than a factor of 3 by \( z = 1 \). The results we obtain are consistent with similar analysis in other papers (Amendola & Quercellini 2003; Jimenez 2003; Jimenez et al. 2003; Padmanabhan & Choudhury 2003; Alam et al. 2004a; Daly & Djorgovski 2004; Barris et al. 2004; Corasaniti et al. 2004; Choudhury & Padmanabhan 2004; Dal...

Let us now add constraints from the WMAP observations. The location of peaks in the angular power spectrum of temperature anisotropies [which depends on the angular diameter distance, which – in turn – depends on $\tau(z)$] provides the main constraint. Other effects such as ISW provide a further constraint but their impact is limited because they are restricted to $l \ll 20$ for most models, whereas we use the full WMAP angular power spectrum ($l \leq 900$). Given that the physics of recombination and evolution of perturbations does not change if we only modify $w(z)$ within some safe limits, any change in the observed angular power spectrum will be due to a change in $w(z)$. The location of peaks in the angular power spectrum of temperature anisotropies will provide the main constraint and hence fix the effective value of $w(z)$. We can define a $w_{\text{eff}}$ by requiring that the angular size of a critical scale like the Hubble radius at the time of decoupling is the same in the model with a given $w(z)$ and for the model where $w(z) = w_{\text{eff}} = \text{constant}$. Observations mainly constrain the value of $w_{\text{eff}}$. Thus if the present value of $w_0 < w_{\text{eff}}$ then we should have $w_1 > 0$, and similarly if $w_0 > w_{\text{eff}}$ then $w_1 < 0$ in the parametrizations we have used. Of course, if $w_1 = 0$ then $w_{\text{eff}} = w_0$.

The priors we have considered here are $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $n = 1$, $\Omega_b = 0.05$, $\tau = 0$, $-1.5 \leq w_0 \leq -0.7$, $-2 \leq w_1 \leq 2$ and $0.2 \leq \Omega_m \leq 0.4$. We do not vary many of these parameters as our main aim is to study the allowed variation in dark energy. If we allow variation of all parameters at once, it will become difficult to understand the impact of individual variations. (We have varied some of the parameters individually, e.g. we have checked that allowing for reionization with a prior $\tau \leq 0.4$ does not make a significant difference to the results, even though the best-fitting value of optical depth $\tau$ is different for different models.) We are carrying out a detailed study where variation of most of these parameters is allowed and we also bring in more constraints (Jassal, Bagla & Padmanabhan, in preparation).

We do not take into account perturbations in dark energy in this paper. Perturbations in dark energy can modify our results to some extent. The effect of perturbations depends on the detailed model of dark energy and cannot be incorporated in the model-independent approach we have used here. Including adiabatic perturbations in dark energy does not change our result significantly, as the effect of these perturbations changes the angular power spectrum only for $l \ll 20$. The effect of perturbations in dark energy becomes more relevant if we consider the allowed values of all cosmological parameters in a varying dark energy model.

To bring in these constraints from WMAP data, we need to compare the angular power spectrum expected in a given model with the observed spectrum (Hinshaw et al. 2003). We compute the theoretical angular power spectrum for each model using cmbfast. The comparison is carried out using the likelihood program made available by the WMAP team (Verde et al. 2003). We normalize the angular power spectrum by maximizing likelihood in a comparison with the WMAP data.

### 3 RESULTS

Analysis of the high-redshift supernova data shows that a large region in the parameter space $\Omega_m - w_0 - w_1$ is allowed at the 99 per cent confidence level. For $w_1 = 0$, the favoured values of $w_0$ decreases (is more negative) with increasing $\Omega_m$. We will show detailed plots only for $\Omega_m = 0.3$ and then discuss the variation of allowed range of parameters with $\Omega_m$. We have studied large regions in parameter space explicitly and have not relied on any expansion around the best fit model.

A fairly rapid evolution in the equation of state is allowed, so much so that $w(z) \geq -1/2$ at $z \geq 0.5$ is consistent with the supernova observations. Fig. 1 shows the allowed region at 99 per cent confidence level in the $w_1 - w_0$ plane for $\Omega_m = 0.3$, regions in the parameter space that are ruled out are blanked out in dark grey (purple in the online version of this Letter). The 99 per cent confidence

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Figure 1. The top panel shows the allowed region at 99 per cent confidence level in the $w'(0) - w(0)$ plane for $\Omega_m = 0.3$ for the parameterization in equation (2) with $p = 1$. The dark grey region (purple in the online version of this Letter) is excluded by supernova observations. WMAP constraints rule out the region in light grey (green) at the same confidence level. The thick dotted line divides models that violate the strong energy condition from those that do not. Models which preserve strong energy condition are to the right top of the dotted lines. The lower panel shows the same plot for $p = 2$. 

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1 http://www.cmbfast.org
limit corresponds to a $\Delta \chi^2 = 11.3$ for the three parameters we have here. These results compare well with earlier studies that have addressed the issue of dark energy. We are not showing the full region allowed by supernova constraints because much of it is ruled out by WMAP observations.

Adding WMAP constraints reduces the allowed region in parameter space in a significant manner. The light grey (green in the online version) region is ruled out by WMAP observations at 99 per cent confidence level in Fig. 1. We have $\Delta \chi^2 = 13.28$ for both $p = 1$ and $p = 2$. The orientation of the confidence level contours is consistent with the assertion that CMB observations essentially constrain $w_{\text{eff}}$. Dotted lines within this region mark the dividing line between models for which $w(z) \geq -1$ at all times (to the right and above the dotted line) and models which have $w(z) < -1$ at some redshift. These lines separate models that violate the condition $(\rho + p) \geq 0$ from those that do not. Equations for these lines are:

\begin{align*}
    & p = 1: \\
    & w(0) = -1 \quad [w'(0) \geq 0], \\
    & w(0) + w'(0) = -1 \quad [w'(0) < 0].
\end{align*}

\begin{align*}
    & p = 2: \\
    & w(0) = -1 \quad [w'(0) \geq 0], \\
    & w(0) + w'(0)/2 = -1 \quad [w'(0) < 0].
\end{align*}

Observations do not discriminate between these two types of models but we have indicated this line for the benefit of those who have a strong theoretical preference.

In Fig. 1, the top panel is for $p = 1$ and the lower panel is for $p = 2$. It is clear that the key features are the same in two panels. For both of the parametrizations the region allowed by WMAP observations is much smaller than that allowed by the supernova observations (and the region occupied by non-phantom models is still smaller). The lower bound from supernovae becomes stronger than the CMB constraint at small $w_0$. The quantitative differences between the figures can be attributed to a difference in $w(z)$ at a fixed $z$ in the two parameterizations.

To get a better feel, we show in Fig. 2 the allowed values of $\rho_{\text{DE}}(z)$ for dark energy as a function of redshift $z$ for $\Omega_{\text{DE}} = 0.3$ with just supernova data and by combining the constraints from WMAP. The dark grey region (purple in the online version) is ruled out by constraints from observations of high-redshift supernovae alone. In cases where the allowed region extended beyond the range of parameters that we have studied, we used the largest allowed value within the range we had studied to get the envelope shown for this plot. For example, the allowed values of $w'(0)$ for supernova constraints go beyond the range that we have studied ($-2 \leq w'(0) \leq 2$). This leads us to underestimate the region allowed by supernova constraints in Fig. 2 but this does not affect the WMAP constraints. In this sense, our main conclusion – that WMAP observations put stronger constraint on $w(z)$ – is better than it appears from this figure. The allowed variation in $\rho_{\text{DE}}(z)$ is slower than that for matter, but considerable evolution is allowed by the supernova observations. The light grey region (green in the online version) is excluded by WMAP constraints. For clarity, we have only marked those regions that are not already excluded by the supernova observations. Models that never have $w(z) < -1$ live between the broken lines. Models that are above the upper broken line are typically those that have $w(0) < -1$ but a positive $w'(0)$. Of course, a model that lives in the allowed region but has a different functional form for $w(z)$ from that in equation (2) may not be allowed. The results for two parameterizations are very similar for $z \lesssim 2$.

4 CONCLUSIONS

The observations we have considered constrain distances and the main constraint from CMB observations is on a reduced quantity $w_{\text{eff}}$, which is the integrated effective value for the angular diameter distance. If $w(0) < w_{\text{eff}}$ then $w'(0) > 0$ is expected, indeed this explains the orientation of the allowed region in Fig. 1. There is a strong degeneracy between $w_{\text{eff}}$ and $\Omega_{\text{DE}}$; $\partial w_{\text{eff}}/\partial \Omega_{\text{DE}} < 0$ (see Fig. 3). The cosmological constant itself is ruled out at the 99 per cent confidence level for $\Omega_{\text{DE}} > 0.375$ and only phantom models survive beyond this, thus if other observational constraints were to rule out $\Omega_{\text{DE}} \lesssim 0.375$ then we will be
This effect is relevant only at approaches zero, leading to suppression of perturbations in matter in dark energy. This effect becomes increasingly important as Bagla & Padmanabhan, in preparation.

Other parameters will be explored in detail in a later work (Jassal, consistent with observations. The detailed dependence on ing dark energy or phantom models, even though such models are constant model is allowed. Hence observations do not require vary-

WMAP constraints on evolution of dark energy

Figure 3. The figure shows confidence levels in $\Omega_{m} - w_0$ plane for supernova and WMAP constraints (for both parametrizations) with $w'(z = 0) = 0$. 99 per cent confidence levels are plotted for both observational constraints. The bold solid contour is the 99 per cent confidence level for WMAP. It is clear that the region allowed by the combination of two observational constraints is much smaller than the one allowed by supernova data.

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REFERENCES

WMAP data and observations of high-redshift supernovae. Indeed, given the allowed window as seen in Fig. 2, the cosmological constant seems to be the most attractive dark energy candidate.

At the present time we are working with parametrized variations. We are carrying out a more detailed analysis as it requires a detailed model for dark energy whereas we are working with parametrized variations. We are carrying out a more detailed analysis where we allow parameters such as $n, h, \Omega_B, \tau$, etc. to vary. Preliminary results suggest that after marginalizing over other parameters the region in the $w(0) - w'(0)$ allowed by CMB observations increases, mainly on the lower side of the region shown in Fig. 1. Supernova observations then provide stronger constraints for smaller $w'(0)$ while the CMB observations continue to provide much stronger constraints on large $w'(0)$. Adding other observational constraints from structure formation reduces the allowed region significantly, as extreme variations of parameters allowed by WMAP data predict too much or too little structure formation (Jassal et al. 2004). These observations are not very relevant in the present work as we are keeping most parameters fixed.

Our analysis of models and observations shows that the allowed variation of $\rho_{DE}(z)$ is strongly constrained by a combination of
Ostriker J. P., Steinhardt P. J., 1995, Nat, 377, 600
Peebles P. J. E., Ratra B., 2003, Rev. Mod. Phys., 75, 559

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