Spinor driven inflation

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Indo-UK meeting © IUCAA 11 August 2011
Theoretical problems of inflation

- Inflation has several problems including:
  - Reheating problem
  - Hierarchy problem
  - Trans-Planckian problem
  - ... 

- These problems seem to be related to the fundamental question:
  What is the nature of the field which drives inflation?
Inflationary models

- Usually assumed to be a scalar field with a potential

![Diagram](image-url)

Potential Energy

Vacuum Energy

Inflation

\[ V(\phi) \]

\[ \varphi \]
Inflationary models

- Is inflaton a fundamental scalar field? Not clear

Effective scalar field of several fields [Nibbelink and Tent, 2001]

How many scalar fields are required?

Higher Ricci scalar curvature terms $R + \alpha R^2$ [Starobinsky, 1980]

Very specific to Ricci scalar; not if Ricci/Riemann tensors are included do not have a well defined initial value problem

Vector field [Golovnev et al '08; Dimopoulos et al '09] leads to directional asymmetry; require infinite of them to restore symmetry; has instabilities
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  leads to directional asymmetry;
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Constraints from current CMB observations

- WMAP-5 data \[ \frac{\delta \rho}{\rho} \simeq 5 \times 10^{-5} \] \[ k_1 = 0.002 \text{Mpc}^{-1} \]

- \( n_s \simeq 0.96 \)

- \( \frac{dn_s}{d \ln k} \simeq -0.037 \)

- \(-9 < f_{NL} < 111\)

- Physical consequence
  - perturbation theory is valid
  - Broadly consistent with inflationary paradigm

However, ...

Canonical single scalar field inflation predicts no running and tiny \( f_{NL} \).

Need to go beyond and look for other alternatives.
Spinor driven inflation
**Basic idea**: Free spinors form an highly interacting Bosonic condensate which dominates in early universe.
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Can such a condensate form in the early universe?

The transition from the free fermions to a highly interacting Bosons occurs below the critical temperature:

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In the early universe, \( \rho \sim 10^{98}\text{g/m}^3 \sim 10^{74}\text{GeV}^4 \), \( m \sim 10^{15}\text{GeV} \)

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What kind of spinors can form such a condensate?
Elkos

- Elkos \( \equiv \) Eigenspinoren des Ladungskonjugationsoperators
  Eigen spinors of charge conjugation operator.
In 1928, Dirac formulated wave-equation for charged spin 1/2 particles.

essentially Dirac wanted to compute the square root of the Klein-Gordon equation \((\partial^2 + m^2)\phi = 0\); using matrix valued objects \((i\gamma^a \partial_a + m)(i\gamma^a \partial_a - m)\psi = 0\)

What kind of spinors are used in the Dirac equation?

Eigen spinors of parity operator
How does one describe a neutral spin 1/2 particle? **Majorana particle**

Under charge conjugation operator, the usual set of two Majorana spinors have eigenvalue one.

Ahluwalia & Grumiller showed that there also exists anti self-conjugate set

Complete set of four spinor (Elko) span the four-dimensional representation space of spin 1/2 and come to par with Dirac spinors

Elko are the eigen spinors of charge conjugation operator
Dirac spinors ($\psi$) 

Elkos ($\lambda$)
<table>
<thead>
<tr>
<th>Dirac spinors ($\psi$)</th>
<th>Elkos ($\lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>eigen spinor of parity operator</td>
<td>charge conjugation operator</td>
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$$\hat{P}|\psi\rangle = p|\psi\rangle$$

$$\hat{C}|\lambda\rangle = c|\lambda\rangle$$
**Elkos**

**Dirac spinors ($\psi$)**
- Eigen spinor of parity operator
  $$\hat{P}|\psi\rangle = p|\psi\rangle$$
- Form of spinor and conjugate
  $$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \quad \psi^\dagger = \gamma^0 \bar{\psi}$$
  8 real, independent functions

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### Elkos

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- 8 real, independent functions
- Satisfy \((CPT)^2 = I\)
- Dirac Lagrangian
  \[ \mathcal{L}_{\text{Dirac}} = \overline{\psi} (i\gamma^\mu \partial_\mu - m) \psi \]

### Elkos ($\lambda$)
- Charge conjugation operator
  \[ \hat{C} | \lambda \rangle = c | \lambda \rangle \]
- Form of Elko and conjugate
  \[ \lambda = \begin{pmatrix} \sigma_2 \phi_1^* \\ \phi_1 \end{pmatrix}, \quad \lambda^\dagger = i \begin{pmatrix} \phi_2^\dagger & \phi_2^\dagger \sigma_2 \end{pmatrix} \]
- 8 real, independent functions
- Satisfy \((CPT)^2 = -I\)
- Elko Lagrangian
  \[ \mathcal{L}_{\text{elko}} = \frac{1}{2} \mathcal{D}_\mu \lambda^\dagger \mathcal{D}^\mu \lambda - m^2 \lambda \lambda^\dagger \]
Standard matter particles satisfy $(CPT)^2 = 1$.
Elkos satisfy $(CPT)^2 = -1$. 
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Mass dimension of Elkos also restrict the kind of interactions.
Mass dimension and interactions

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  Interactions between Elkos and standard matter particle will always need Elko and its conjugate.

- Mass dimension of Elkos also restrict the kind of interactions.

Unlike Dirac fields, Elkos can ONLY interact with standard matter particles via Higgs and/or gravity.
Consider the following \((3 + 1) - d\) action

\[
S = \int d^4x \sqrt{-g} \left( R + \mathcal{L}_{\text{Elko}} \right)
\]

\[
\uparrow
\]

\[
\frac{1}{2} \left[ \frac{1}{2} g^{\mu\nu} (\mathcal{D}_\mu \lambda^\dagger \mathcal{D}_\nu \lambda + \mathcal{D}_\nu \lambda^\dagger \mathcal{D}_\mu \lambda) \right] - V(\lambda^\dagger \lambda)
\]
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FRW line-element:

\[ ds^2 = dt^2 - a^2(t) d\tilde{x}^2 = a^2(\eta) \left[ d\eta^2 - d\tilde{x}^2 \right] \]

\[ \uparrow \quad \uparrow \quad \uparrow \]

cosmic time expanding 3-space conformal time
Effective density and pressure

\[ \rho = \frac{1}{2} \frac{(\varphi'(\eta))^2}{a^2(\eta)} + V(\varphi) - \frac{3}{8} \frac{\mathcal{H}^2}{a^2(\eta)} \varphi^2(\eta) \]

\[ p = \frac{1}{2} \frac{(\varphi'(\eta))^2}{a^2(\eta)} - V(\varphi) + \frac{1}{8} \frac{\mathcal{H}^2}{a^2(\eta)} \varphi^2(\eta) \]

\[ \mathcal{H} = \frac{a'}{a} \]

extra terms
Effective density and pressure

\[ \rho = \frac{1}{2} \left( \frac{\varphi'(\eta)}{a^2(\eta)} \right)^2 + V(\varphi) - \frac{3}{8} \frac{\mathcal{H}^2}{a^2(\eta)} \varphi^2(\eta) \]

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\[ \rho + 3p = 2 \left[ \frac{(\varphi'(\eta))^2}{a^2(\eta)} - V(\varphi) \right] \quad \Rightarrow \quad \text{Identical to canonical scalar field} \]
FRW background

Boehmer '07 '08, SS '09, Gredat & SS '10

- Effective density and pressure

\[ \rho = \frac{1}{2} \left( \varphi'(\eta) \right)^2 + \frac{3}{8} \frac{H^2}{a^2(\eta)} \varphi^2(\eta) \]

\[ p = \frac{1}{2} \left( \varphi'(\eta) \right)^2 - \varphi(\varphi) + \frac{1}{8} \frac{H^2}{a^2(\eta)} \varphi^2(\eta) \]

Extra terms

- Acceleration equation is identical to canonical scalar field driven inflation

\[ \ddot{a} = -\frac{4\pi}{3M_{Pl}^2} (\rho + 3p) = \frac{8\pi}{3M_{Pl}^2} \left[ V(\varphi) - \dot{\varphi}^2 \right] \]

\[ M_{Pl} \equiv G^{-1/2} \approx 10^{19}\text{GeV} \]

Impossible to distinguish the two models from the acceleration equation.
Modified Friedman equation

\[
\frac{8\pi}{3M_{Pl}^2} \left[ V(\varphi) + \dot{\varphi}^2/2 \right] \left[ 1 + F \right] = H^2
\]

\[
\ddot{\varphi} + 3H\dot{\varphi} + G(\varphi) + V,\varphi = 0
\]

\[
F = \frac{\varphi^2}{8M_{Pl}^2}
\]

Salient Features:

1. Elko (and its dual) depends on a single scalar function (\(\varphi\))

   Physically, this can be interpreted an Elko-pair (similar to Copper-pair) forming a scalar condensate — spinflaton.

2. Friedman and spinflaton equations receive non-trivial corrections

   Elko modification to the inflaton equations are determined by \(F\)
Slow-roll parameters

Exact de Sitter solution

\[ V(\varphi) = 3q^2 M_{Pl}^2 + \frac{q^2}{4} \varphi^2 \]

\[ \varphi \propto \exp \left( \pm \frac{qt}{2} \right) \]

\[ a \propto \exp(qt) \]

Different from the canonical scalar field

[Boehmer '08]
Slow-roll parameters


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- First order exact slow-roll parameters are:

\[ \varepsilon \equiv -\frac{\dot{H}}{H^2} = \varepsilon_{\text{can}}[1 + \mathcal{F}] - \mathcal{F} \]

\[ \delta \equiv -\frac{\ddot{\varphi}}{H \dot{\varphi}} = \delta_{\text{can}} + \mathcal{F}(\varepsilon_{\text{can}} - 1) - \frac{\ln(1 + \mathcal{F})'}{2H} \]

\[ \varepsilon_{\text{can}} = 3 \frac{\dot{\varphi}^2/2}{\dot{\varphi}^2/2 + V} \]

\[ \delta_{\text{can}} = \varepsilon_{\text{can}} - \frac{\dot{\varepsilon}_{\text{can}}}{2H \varepsilon_{\text{can}}} \]
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- Slow-roll approximation corresponds to

\[ \varepsilon, \delta \ll 1 \quad \implies \quad \varepsilon_{\text{can}}, \delta_{\text{can}} \ll 1, \mathcal{F} \ll 1 \]
Consider small inhomogeneities:

\[ \lambda = \bar{\lambda} + \delta \lambda \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \quad \delta g_{\mu\nu} = \delta g_{\mu\nu}^{(S)} + \delta g_{\mu\nu}^{(T)} \quad \left| \frac{\delta g_{\mu\nu}}{g_{\mu\nu}} \right| \ll 1 \]

\[ \delta g_{\mu\nu}^{(S)} = a^2(\eta) \begin{pmatrix} 2\Phi & 0 \\ 0 & -2\Psi \delta_{ij} \end{pmatrix} \quad \delta g_{\mu\nu}^{(T)} = a^2(\eta) \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix} \]
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Scalar and tensor perturbations decouple; can be treated separately.
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- Scalar and tensor perturbations decouple; can be treated separately
- Elkos do not source the tensor perturbation equations and they are free gravitational waves:

\[ \mu_T'' + \left( k^2 - \frac{a''(\eta)}{a(\eta)} \right) \mu_T = 0 \]
Issues

- Scalar perturbations are harder to compute even for the scalar fields which have one free real function.
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- Elkos have 8 real functions and not all are independent.

Such an analysis has not be done for any spinor in the literature!
Linear perturbation

**Approach**

- Assume the anisotropic stress of the perturbed Elko is zero

\[ \Phi \rightarrow \Psi \implies \delta T_{ij} = 0 \quad \forall \quad i \neq j \]
**Linear perturbation**

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- Perturbed Elko must satisfy \([\delta \varphi(x) \text{ is perturbed condensate}]\)

\[ \overline{\lambda}^\dagger \delta \lambda + \delta \lambda^\dagger \overline{\lambda} = 2 \overline{\varphi} \delta \varphi \]
Scalar perturbation equation is

\[ \mu''_S - \left[ -k^2 + \frac{z''}{z} - \ln[1 - \mathcal{F}_\epsilon]' + \frac{7 \mathcal{H}' \mathcal{F}_\epsilon^{\frac{1}{2}}}{2} + \frac{\mathcal{H}_\epsilon \mathcal{F}_\epsilon^{\frac{1}{2}}}{\epsilon} \right] \mu_S = 0 \]

Different from the canonical scalar field
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Different from the canonical scalar field

**Salient Features**

- Elko modification to the canonical MS equation is determined by \( F \).
- This equation is exact.
Upon quantization, in the slow-roll limit $\epsilon, \delta \ll 1$, power-spectra are:

\[
\mathcal{P}_S(k) \simeq \left( \frac{H^2}{8 M_{Pl}^2 \pi^2} \right) \left( \frac{\epsilon + \mathcal{F}}{\epsilon^2} \right) \left[ 1 - 2(c_0 + 1)\epsilon_{\text{can}} \right]
\]

\[
\mathcal{P}_T(k) = \left( \frac{2H^2}{M_{Pl}^2 \pi^2} \right) \left[ 1 - 2(c_0 + 1)\epsilon_{\text{can}} + 2\epsilon_{\text{can}} x \right]
\]
Results and implications
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\( \mathcal{P}_S(k), \mathcal{P}_T(k), \) during slow-roll, are nearly scale-invariant
Results and implications

- $P_S(k), P_T(k)$, during slow-roll, are nearly scale-invariant.

- Predicts running of spectral index at the leading order of $\epsilon$

\[
\frac{dn_s}{d \ln k} = -\frac{\epsilon_{\text{can}}}{2} - 4\epsilon_{\text{can}} \mathcal{F}_\epsilon^{1/2} + \frac{\epsilon_{\text{can}}}{2} \frac{\mathcal{F}}{1 + \mathcal{F}}
\]

\[
\frac{dn_T}{d \ln k} = 2\epsilon_{\text{can}} \mathcal{F}_\epsilon^{1/2}
\]

Consistent with WMAP data.
Results and implications

- \( \mathcal{P}_S(k), \mathcal{P}_T(k) \), during slow-roll, are nearly scale-invariant

- **Modified consistency relations**: Scalar and tensor perturbations originate from the scalar condensate and they are not independent. Consistency relations link them.

  1. Tensor-to-scalar ratio is \( r \approx 16 \varepsilon_{\text{can}} [1 - 2 \mathcal{F}_\varepsilon] \)

  Tensor contribution is smaller compared to canonical inflation

  2. The other observationally useful is the relation between \( n_T \) and \( r \):

\[
n_T = \frac{r}{8} (1 + \mathcal{F}_\varepsilon) \left[ 1 + \varepsilon_{\text{can}} \left[ \frac{11}{6} c + \mathcal{F}_\varepsilon - \mathcal{F} \right] - 2 \delta_{\text{can}} c \right]
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     Different from the scalar field inflation
Conclusions

Fermions forming a scalar condensate is a real alternative to the scalar field model of inflation.

- It leads to attractor behavior [Basak, Bhat & SS ’11]

- For the first time perturbation equations for a spinor field are derived

- Scalar condensate from Elkos lead to observationally consistent primordial power spectra

- Predicts running of spectral index and modified consistency relations
Issues

- As in the canonical scalar field the form of the potential is unclear.
- Power spectra calculations relies on the slow-roll condition.

Outlook

- Can it lead to large non-Gaussianity? [Basak & SS, Work in progress]
- Can Elko condensate lead to growing vorticies and hence magnetic field? [Work in progress]
Propagator of the Elko field is

\[ G^{\text{Elko}} = \int d^4 p \frac{1}{(2\pi)^4} \exp^{-ip_{\mu}(x'^{\mu} - x^{\mu})} \frac{\mathbb{I}}{p^{\mu}p_{\mu} - m^2 + i\epsilon} \]

Compare this with the propagator of the Dirac spinor

\[ G^{\text{Dirac}} = \int d^4 p \frac{1}{(2\pi)^4} \exp^{-ip_{\mu}(x'^{\mu} - x^{\mu})} \frac{\gamma^{\mu}p_{\mu} + m\mathbb{I}}{p^{\mu}p_{\mu} - m^2 + i\epsilon} \]

Mass dimension of Elkos is different from Dirac spinors while it is same as Klein-Gordon field
Form of Elkos in the background

- Form of Elko which leads to $T_{ti} = T_{ij} = 0$: 

$$
\lambda = \frac{\overline{\varphi}(t)}{\sqrt{12}} \begin{pmatrix}
-\alpha_1 e^{i\pi/4} \\
\alpha_2 \frac{i}{\sqrt{2}} \\
\alpha_2 \frac{1}{\sqrt{2}} \\
\alpha_1 e^{i\pi/4}
\end{pmatrix}
\quad \lambda^\dagger = \frac{\overline{\varphi}(t)}{4\sqrt{12}} \begin{pmatrix}
-\alpha_1 e^{-i\pi/4} & -i\alpha_2 & \alpha_2 & \alpha_1 e^{-i\pi/4}
\end{pmatrix}
$$

$$
\alpha_1 = \alpha_2^{-1} = \sqrt{1 + \sqrt{3}}
$$
Scalar perturbation equations

\[ \Delta \Psi - 3 \mathcal{H} \Psi' - (\mathcal{H}' + 2 \mathcal{H}^2[1 + F(\varphi)]) \Psi = 0 \]

\[ = \frac{1}{2 M_{\text{Pl}}^2} \left[ \varphi' \delta \varphi' + a^2 V_{,\varphi} \varphi \right] + 3 F(\varphi) \mathcal{H} \left[ \Psi' - \frac{\mathcal{H}}{\varphi} \delta \varphi \right] \]

\[ \Psi' + \mathcal{H}[1 + F(\varphi)] \Psi = \frac{1}{2 M_{\text{Pl}}^2} \varphi' \delta \varphi \]

\[ \Psi'' + 3 \mathcal{H} \Psi' + (\mathcal{H}' + 2 \mathcal{H}^2[1 + F(\varphi)]) \Psi = 0 \]

\[ = \frac{1}{2 M_{\text{Pl}}^2} \left[ \varphi' \delta \varphi' - a^2 \frac{V_{,\varphi}}{2} \delta \varphi \right] - F(\varphi) \mathcal{H} \left[ \Psi' - \frac{\mathcal{H}}{\varphi} \delta \varphi \right] \]

\[ \delta \varphi'' - \Delta \delta \varphi - \varphi' \left[ 4 - 3 \left( 1 - \varepsilon \right) F_{,\varepsilon} - 3 \sqrt{F_{,\varepsilon}} \right] \Psi' + \mathcal{H} \left[ 2 + 3 \left( 1 - \varepsilon \right) F_{,\varepsilon} + 2 F \right] \delta \varphi' \]

\[ + a^2 \left[ V_{,\varphi} \Psi + \frac{1}{2} V_{,\varphi \varphi} \delta \varphi \right] - \frac{3}{4} \mathcal{H}^2 \left[ 1 - \frac{8}{3} F \left[ 3 + \frac{\mathcal{G}}{\mathcal{H} \varphi'} - \delta \right] + 4 \left( 1 - \varepsilon \right) \sqrt{F_{,\varepsilon}} \right] \delta \varphi \]

\[ - 2 \mathcal{H} \varphi' \left[ 3 + \frac{\mathcal{G}}{\mathcal{H} \varphi'} - \delta \right] \Psi + \frac{2}{\sqrt{3} \mathcal{H}} \varphi' F_{,\varepsilon} \nabla \Psi' = 0 \]
Mukhanov-Sasaki \((Q)\) variable

- \(Q\) is a gauge-invariant linear combination of \(\delta \varphi\) and \(\Psi\)

  Also related to the curvature perturbation \(\mathcal{R}\)

- Unlike scalar field, not possible to obtain \(Q\) directly from \(\delta \varphi\) and \(\Psi\)

  \(\delta \varphi\) is derived from \(\delta \lambda\)

- **Approach:** Assume the relation between \(\mathcal{R}\) and \(Q\) is like that of canonical scalar field.

- This leads to

\[
Q = a \delta \varphi + z \Psi \quad \quad z = [1 - \mathcal{F}_\varepsilon] (a \varphi') / \mathcal{H}
\]
Scalar perturbation equations

Salient Features

- Matter perturbations have one dof — perturbed condensate

Consistent linear perturbation equations
Scalar perturbation equations

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- Matter perturbations have one dof — perturbed condensate

  Consistent linear perturbation equations

- Sound speed of perturbations is 1 \( c_s^2 = 1 \)
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- Entropic perturbations vanish at super-Hubble scales

\[ \delta S \propto \nabla^2 \psi \]
Scalar perturbation equations

**Salient Features**

- Matter perturbations have one dof — perturbed condensate

**Consistent linear perturbation equations**

- Sound speed of perturbations is 1 \( c_s^2 = 1 \)

- Entropic perturbations vanish at super-Hubble scales

\[ \delta S \propto \nabla^2 \Psi \]

- Curvature perturbation \( \zeta \) is given by

\[ \zeta = \Psi + \mathcal{H} \frac{\delta \varphi}{\varphi'} \frac{1}{(1 - F_\epsilon)} \]