Hello everyone. Welcome to the third lecture on cosmology. So in the last lecture we talked about the FLRW metric and how to write the Hubble parameter for such a Universe. So just to remind you, suppose you want to build up a cosmological model, what would you do? You first list all the matter or energy components that constitute the Universe. For each component $\alpha$ you are supposed to know the value of $w_\alpha$, that is the pressure density relation. You also need to know the corresponding density parameter denoted by $\Omega$. So for $\alpha$ component, the density parameter will be $\Omega_\alpha$. If you evaluate it at the present epoch, we will call it $\Omega_\alpha 0$ as is written here.

Now don't forget to calculate the curvature next because that is crucial. So the curvature parameter will simply be $1$ minus summation over all the components $\Omega_\alpha 0$. So you sum over all the densities, subtract one, the negative of that is the curvature.

Now you are in a position to write the Hubble parameter. So the Hubble parameter is a dot square by a square. So that's the square of the Hubble parameter, that's $H$ square $a$. You write it as $H$ naught square, the present value of the Hubble parameter and then you sum over all the components $\alpha$. What do you do inside? You put the $\Omega_\alpha$ at present epoch and then multiplied by the corresponding time evolution. We have already seen that if the pressure density relation or the equation of state is $w$, then the evolution is given by $a$ to the minus $3(1 + w)$. And that's precisely what is written here.

Then at the end you also add the curvature term that comes with $\Omega k 0$ divided by a square. So the curvature evolves as $1$ by a square. Now, this is a differential equation in $a$ as a function of $t$. So in principle, if you know everything on the right hand side, you can solve for $a(t)$ and then you are in a position to calculate all relevant quantities of interest. You can calculate the luminosity distance, the angular diameter distance or any other cosmologically relevant quantity. These are things which we have already discussed in the previous lecture.
Section 4.3.1
Simple cosmological models

[00:03:06] Now let us so this what we what I talked about in the previous slide was for a general Universe. For any Universe which has various matter on energy components, you can work out the solution. You will write the Hubble parameter and you exactly know what to do. Now let us take some special examples which are interesting. So let us first consider the case where the Universe is spatially flat. So we have restricted ourselves to k equal to 0 and let us for simplicity assume that the Universe is filled with only one kind of matter and that matter has an equation of state given by w, as is written here. Of course in such a Universe, we know that the density of matter will simply evolve as ρ equals to ρ naught the present value times a to the minus 3 1 plus w. Again, this is something which we have done earlier. Also we know since the Universe is flat, the Ω must be one so that the curvature term goes to zero.

[00:04:18] Now we are in a position to write the Hubble parameter. So the Hubble parameter square is just H naught square and then we said that we will sum over all the components. Here, it is just one component, that too has Ω equals to 1. So that is the only term which comes. If you wanted to put curvature there would be another term corresponding to curvature. But since we are working in a flat Universe that curvature term is not there anymore. This differential equation can now trivially be solved. If you solve it, you will get a going as t to the power 2 divided by 3 1 plus w. This is a trivial exercise which one can do. Of course this solution holds only when w is not equal to minus 1.

[00:05:07] Remember this was the case where we didn't specify what w is. So this solution holds for any w. So let us now consider few special cases of this. For example, if the Universe is dominated by non-relativistic matter, so there is just one type of matter and that too is non-relativistic. We saw that for non-relativistic matter, the pressure is negligible so we can safely put w to be zero. So if you put w to 0 in this equation, you will find that the scale factor simply evolves as t to the two third. So Universe filled or dominated by non-relativistic matter will have this kind of evolution where the scale factor will evolve as time to the power two third. A flat matter dominated Universe has a name, it's usually called the Einstein-de Sitter Universe.

[00:06:07] We can go to the next case where we are talking about a flat radiation-dominated Universe. In that case, of course, we have to put w equals to one third. If you put w equals to one third in the solution we obtained above, you will find that a is simply proportional to square root of t. So now you can see, you are now in a position to solve for cosmological models the moment somebody tells you or you know what are the constituents of that model or what are the constituents of the Universe?
You can calculate many things more. Okay. So for example, you can calculate the age of the Universe. In the last lecture, we have already written down an expression for the age in terms of Hubble parameter. If you solve for this Universe, you will find that the age is given by two thirds divided by 1 plus $w_H^{-1}$. This actually confirms the fact that the inverse of the Hubble parameter is actually a very good estimate of the age. To the order of magnitude it will tell you what the age of the Universe is.

Now also note that if we look at the second Friedmann equation, which gives you the acceleration, so the acceleration is as you remember contributed by not only density but also three times the pressure. So if you write it, you will get an equation which will contain a term like $1 + 3w$, 3 times the equation of state. Now, you already know how $\rho$ evolves with $a$, so you can solve it to find the what will be the acceleration as a function of the scale factor, which is simply written here.

The thing to note is the acceleration comes with a negative sign. Everything else here is expected to be positive, for example for non-relativistic matter and radiation you can put $w$ equal to 0 here or $w$ equal to one third here, the right hand side overall will still be negative. So this tells us that acceleration will be negative for non-relativistic matter and radiation. Hence, the Universe will decelerate whenever it is filled with matter of this kind ok, normal matter, which we know of.

In fact, if you want to make the Universe accelerate you need a $w$ which has to be less than minus of one third only then the term in this brackets will become negative and so the right-hand side overall will become positive. So for acceleration, you need a matter which has $w$ equal to minus one third, if the density is positive this implies that you need a matter which has negative pressure. Clearly we don't know of any such matter in laboratory and I will come back to this point later in more details.

You can also for example relate the age of the Universe to its density. Since you have already solved for the density as a function of $a$ and now you also know as a function of $t$, there is nothing which can stop you from writing $\rho$ as a function of $t$. And that is what I have done here. So at the end you will find that $\rho$ evolves as $1/t^2$ with the proportionality factor depending on $w$. So no matter whether you are filled with radiation or matter, $\rho$ will evolve as $1/t^2$. It's only the proportionality constant which will change.

Now let us take another example. In this example, what we assume is that the Universe is filled with non-relativistic matter. So it is a matter-dominated Universe with $w$ equal to 0. But in this case, we allow curvature to be nonzero. So it is matter dominated but it can have curvature if it wants. Again now, you can simply write the expression for the Hubble parameter. So $H^2$ will be $H_0^2$ naught square and then you have to write the term
coming from the matter component. In this case, there is just one type of matter which evolves as 1 by a cube. The density evolves as 1 by a cube. So that is the first term and then you have to put in the curvature term. The curvature term will contain 1 minus \( \Omega \) of that matter or \( \Omega k_0 \) whichever way you want to write, okay.

[00:10:56] Now this equation you can just rewrite in a slightly different form. Remember you have a dot square by a square in the left hand side. So you can write it in a form which is quite suggestive. So you can write it as half a dot square minus \( H_0 \) square \( \Omega_0 \) divided by 2a, which is this term taken to the left hand side and appropriately multiplied by a square. And that is equal to the curvature term which remains on the right hand side. Okay, remember I have multiplied this equation by a square.

[00:11:30] Now this equation should be familiar to you at least mathematically. This is the energy equation for a particle moving in a 1 by r square gravitational field. This will be the half \( v^2 \) term, the kinetic energy term. That is the this is the potential energy term which goes as 1 by r. And in the right hand side, you have a constant which is like the total conserved energy. So mathematically the Universe is just like you have thrown a stone from the surface of the Earth and depending on what energy you give, the trajectory of that stone behaves in a certain way.

[00:12:10] For example, if the energy term is positive, which means \( \Omega k_0 \) is positive means \( \Omega_0 \) the \( \Omega \) of matter is less than 1. Then we know that the motion will be unbounded, if you throw a stone it will never come back. In this case what it means is that the Universe will expand forever. It will never kind of start that it will never start contracting. So that will happen when \( \Omega \) is less than 1 and this is corresponds to the fact that there is simply not enough matter to halt the expansion.

[00:12:52] The in the other case, when the energy is negative, that will be the case where matter density is greater than 1. In this case, again remember the example of the stone, if you throw a stone with energy less than 0, the stone will go up for some time. It will reach a maximum height and then it will come back. Same thing will happen to the Universe, it will expand it will reach a maximum size and then it will start contracting. So that is the case when you have \( \Omega \) greater than 1. The flat Universe corresponds to the case, which is exactly in between. This is the case when the particle moves exactly with the escape velocity in our analogy. So in this case, the Universe will keep on expanding asymptotically but just about.

[00:13:48] This picture actually shows the behavior of the scale factor as a function of time for matter-dominated Universe, in these three cases. So let us take the case of the closed Universe first. Closed Universe means you have more matter, the \( \Omega \) is greater than 1. In this case, you should follow the blue curve. You can see the scale factor increases, reaches a
maximum and then it starts contracting and it hits zero. So the Universe re-collapses to a point. In fact, you can exactly calculate at what time it will re-collapse and that time is given by expression which I have written here. So the time of collapse is essentially this and that depends on how much matter you have put in. If you put more matter, it will collapse faster and vice versa.

[00:14:44] The other extreme is the open Universe. In this case, \( \Omega \) is great less than 1 and so it will never contract, it will keep on expanding and that is shown by the green curve. The one at the boundary of these two cases is a flat Universe. Of course remember in the flat Universe, we have already solved it explicitly. The scale factor actually evolves as \( t \) to the two third and that is given by the orange line.

[00:15:13] So this should give you an idea that again, once you know what matter densities are there in your cosmological model, you can solve it, you can calculate how the Universe expands or contracts or accelerates or decelerates or whatever and you can calculate all other quantities of interest.

Section 4.3.2
Constituents of our Universe

[00:15:39] Now you can ask the question that, okay, we have worked some examples but what is the case in our Universe? What are the matter densities in our Universe? What are the different type of constituents which arise there? And what are the corresponding density parameter? Basically, we now want to solve for the scale factor or the Hubble parameter for the Universe we live in. So to do that as remember, we just first have to list all the components with their \( \Omega \) parameters.

[00:16:14] So it turns out, depending on what we know now the main components of our Universe are radiation, ordinary matters baryons, okay, so this should be a bracket which needs to be closed. Sorry for the typo here. There will be dark matter and I will tell you what dark matter is very soon and there is something called dark energy. So as per our understanding the main components of our Universe are these four okay.

[00:16:44] So let us take them one by one. Let us start with radiation. A bit of history here, so in 1965 two scientists in the Bell Laboratory in the U.S., Penzias and Wilson, they had actually built a radio as instrument to do some radio astronomy and some satellite communication experiments. And then when where they were doing these trying to do this experiments rather, they found that there is some excess radio signal which is captured by their instrument and they could not account for that. This radio signal came from all over.
the space. This was kind of direction independent and even after they had accounted for every known source of radiation, this thing could not be explained.

[00:17:31] In fact, it turns out that in 1948, well before these experiments was going on, three scientists Gamow, Alpher and Herman had actually theoretically predicted the existence of a background radiation called the cosmic background radiation, which is a natural outcome from Big Bang. Basically it originated when the Universe was small and hot and then as the Universe expanded, it cooled down. Remember the temperature simply goes down with the scale factor as the Universe expands. This theory in fact helped in interpreting the signal Penzias and Wilson were finding.

[00:18:14] So basically, I mean accidentally though Penzias and Wilson what they discovered was a radiation which is often popularly called as left over from Big Bang. So it is coming from the early Universe. It has cooled down because of the expansion of the Universe and that's basically a cosmological signal. So that is a radiation, that's a bulk of the radiation which constitutes our Universe. So Penzias and Wilson, these are their pictures. They actually were awarded the Nobel Prize in 1978 for the discovery of the cosmic microwave background.

[00:18:53] Now I will describe the cosmic microwave background in more details probably in the next lecture. But essentially the prediction of the Big Bang model is that because the photons were constantly interacting or getting scattered of free electrons in the early Universe, the radiation will have a very perfect blackbody Spectrum. Now is this is this the case for the radiation observed by Penzias and Wilson?

[00:19:27] So it's indeed in 1990s a satellite were sent up. It is named COBE. It did many things for the cosmic microwave background, but in particular it may measure the spectrum. In other words, it measured the intensity of this radiation as a function of frequency. What did they get? What they got was what is shown in this picture. So the y axis is that intensity, x axis is frequency. The point with error bars are the data, except that the error bars are expanded or increased artificially 400 times. Otherwise, you wouldn't be able to see it in this picture.

[00:20:17] The solid curve is a blackbody curve which has a temperature of 2.725 Kelvin, the intensity corresponding to a black corresponding to a blackbody radiation. You can see this solid curve is a perfect perfect fit to the data. So that tells us that the cosmic microwave background is a blackbody radiation to a very very very high degree of accuracy. I think this is the perfect blackbody known in the Universe and we can actually measure its temperature very accurately. This also called for a Nobel Prize. So because of the COBE experiment, these two scientists Mather and George Smoot, they won the Nobel Prize in 2006.
This was a big success for the Big Bang model because this cosmic microwave background radiation, the existence of it is a direct prediction of the Big Bang model. And in fact theories which are alternative to Big Bang, which existed at that time, most of them actually fail to explain this very nice blackbody spectrum. And so slowly they were ruled out and that's why currently the hot Big Bang model is the most successful model for explaining our Universe.

Now, so that this tells us that there is radiation in the Universe, which is great. But now we can ask the question that what is the $\Omega$ corresponding to it? Now that is not very difficult to calculate because we know the temperature of the radiation, so we can calculate the density because it's a blackbody. So the density will just be proportional to $T$ to the 4 and the proportionality constant is well known. So I have just done the calculation here and you can show that it will come out to be somewhere around 10 to the minus 13 ergs per cc. You can convert this into an equivalent mass density, so I have to divide by an appropriate factor of $c^2$. So we will get a value which will look like this.

Of course, what we are interested is in the $\Omega$. So I have to divide this density by the critical density. So the $\Omega$ will turn out to be some 2.45 into remember 10 to the minus 5. So it's a very small $\Omega$. The contribution is very small today, for the radiation. Remember, this is the $\Omega$ evaluated today. It turns out that in addition to radiation, the standard electromagnetic radiation, the Universe consists of relativistic neutrinos as well. So if you just include relativistic neutrinos and just add them up, the value of this $\Omega$ goes up a bit and it will from 2.45, it will go up to somewhere around 4.30. It's never this much much less than 1 but yes, this is the value of $\Omega$ one should deal with it. So that is the first component, the first term which will go in the right hand side of that Friedmann equation or the expression for the Hubble parameter.

Right. Now, what about the second component? We said baryons, that's just ordinary matter. Now this baryons are basically contributed by stars, by gaseous matter between stars, which we call the interstellar medium. Not only that, there is a diffuse medium between galaxies, the intergalactic medium and so on. So anything which is made up of ordinary matter, like electrons, protons, neutrons, anything which we are made up of will contribute to the baryons.

These baryons are mostly non-relativistic at present so they can be treated like non-relativistic matter and present estimates imply that the baryonic component contributes to about 0.04 to 0.05 in the $\Omega$. How is this calculated? I will give you some hints again in the next lecture, but let us for the moment take this value and move ahead.
The next thing which we talked about I listed was dark matter. It's also called cold dark matter for a reason, but let's first understand what is cold dark matter. So the existence of this dark matter was known even before cosmological observations really came up and I will try to give you some simple examples of this. For example, when people were observing clusters of galaxies, for example the Coma cluster, what they could you can apply the virial theorem to this clusters of galaxies.

So the average v square, which is the kind of RMS velocity or the velocity dispersion turns out to be of the order of G times M of the cluster divided by the size of the cluster. It turns out that by measuring the velocities of kind of individual stars, the Doppler shifts of them, you can calculate this velocity dispersion. You can also calculate the size by knowing the distance to the cluster and somehow estimating the angular size or if you some other proxy for size you can use that as well. So if you know v square, the velocity dispersion and the size, you can in principle calculate the mass of the cluster.

On the other hand, you also can calculate the mass contributed by the stars or the gas within the cluster and it turns out that the mass which is estimated by applying virial theorem is 10 times the the visible mass, the mass contributed by gas in this case. So this tells you that there is some mass in the cluster which cannot be observed, it's there because you can feel its gravitational effect. That's what the virial theorem is telling us, but we cannot see them. So that indicated that there must be some matter which has gravitational interaction, but probably no electromagnetic interaction. So that matter at that time was termed as dark matter.

There were other evidences for dark matter which came from what we call the rotation curve of galaxies. So you take galaxies and measure the velocities as a function of distance from the center of the Galaxy. These velocities could be velocities of for example hydrogen atoms, which are rotating in the Galaxy. So take this as the rotational velocity. So a curve like this is called rotation curve. And if you now theoretically model this rotation curve, what you can show is that the velocity should actually fall as square root of R beyond the visible galaxy mass. So once you are out of the Galaxy, you have gone way beyond where the stars are, you expect the velocity to decrease is 1 as 1 by root R.

What you actually observe is this, the velocity goes up which is as what theoretical models would predict. But at this point you expect it to decrease like this, instead it remains more or less flat. So this tells you that there is some matter beyond the visible galaxy. In fact, you probably need a matter which falls as 1 by R square to fit the flat curve. You can do more detailed modelling and find out how what kind of profiles which fit, would fit a particular galaxy. But whatever be the case, it seems that there is matter beyond the visible matter which interacts gravitationally but does not emit any electromagnetic radiation. So again, this gives you a separate indication, a independent indication of dark
matter. The first example was for clusters of galaxies. This is something which has to do with individual galaxies. So outskirts of galaxies.

[00:28:49] Now we have gone way beyond that, the cosmological observations confirm this fact that there is indeed dark matter and not only that, the cosmological observation indicate that this dark matter is non-relativistic. Hence, it's called cold dark matter and the corresponding Ω at present is of the order of somewhere around 30%, 0.27 to be precise.

[00:29:16] At this moment, we don't know what this dark matter is because there are no viable candidates in the static Standard Model of particle physics. The Standard Model of particle physics tells you what all matter in the Universe is made up of, at least that's what we think, but it doesn't give any candidate for the dark matter. So just to summarise what we have done, we have done radiation till now, we have talked about baryons till now, we have also now talked about dark matter. Remember both baryons and dark matter are non-relativistic. So but the dark matter content is much more than the baryons. So the baryons, the ordinary matter we are made up of, gives you somewhere around only four to five percent. Whereas, the cold dark matter component is coming out to be much more, more like 25 to 30 percent. So in that sense, the non-relativistic component of matter in the Universe is highly dominated by this dark matter.

[00:30:21] Now finally, we have to talk about the accelerating Universe because as we have discussed, radiation, this baryons and the cold dark matter, none none of them are going to give you a Universe that is accelerating. All of them will give rise to a Universe that is decelerating. However, we already discussed that the supernova data indicates that the Universe is accelerating at late times.

[00:30:47] So the condition for acceleration is that you have to satisfy a condition which looks like this, that ρ plus 3 P divided by c square has to be less than 0. If you now assume that P as P is related to ρ by a equation like this, which we have been assuming so far. And if you assume that the density cannot be negative, density has to be positive definite, then what you find is that the pressure has to be negative and the w has to be less than minus one third. Such matter which is never seen in the laboratory, we don't know what it is. And that's now called dark energy. So now we believe that the Universe has a significant amount of dark energy so that it can give rise to acceleration.

[00:31:39] In fact, if we analyze the data very carefully, the supernova data combined with other data sets, what we find is w is not only less than one third but actually w is very close to minus 1. Now if w is minus 1, what you can do is you can solve the conservation equation and show that the density has to be constant. That is, the density does not decrease even when the volume expands which is what you would expect for normal matter.
The dark energy component which has $w$ equal to minus 1 and $\rho$ equal to constant is known as the cosmological constant and is denoted as this $\Lambda$. Okay, so the Universe has a cosmological constant and that is the main reason why it is expanding. According to present estimates, the $\Omega$ corresponding to $\Lambda$, which of course is not evolving is of the order of 70 percent, much more than any other component we have discussed so far.

This is the kind of cosmic pie. It tells you how what are the various constituents of matter in the present-day Universe. As you can see, the pie is dominated by dark energy, somewhere around 68 percent. Ordinary matter is very less, somewhere around 5 percent. The dark matter is somewhere around 27 percent. So these are the things which we already discussed. Remember dark matter plus ordinary matter constitutes the non-relativistic component of matter. You may ask where did the radiation go? The radiation is negligible. So even if we try to plot it here, it won't show up.

So the present-day Universe has radiation, although it has this cosmic microwave background all over, the net contribution is really negligible as far as this ultra relativistic or radiation component is concerned. What you could also see that if you add up the omegas for this these components, the $\Omega$ turns out to be very close to 1. So again observations indicate that we are living in a Universe, which is very very very close to flat. We are probably living in a spatially flat Universe.

Section 4.3.3

Standard model of cosmology

So now we know that the constituents of the Universe and the corresponding Omegas. So we may as well write the expression for Hubble parameter. So Hubble parameter, since there are four components, radiation, baryons, dark matter and dark energy, we expect the Hubble parameter to contain these four terms. However, what happens is both dark matter and baryons, since both are non-relativistic, both of them evolve as $1/a^3$. So we may as well add them up. So we call it $\Omega$ matter which is nothing but $\Omega$ dark matter plus $\Omega$ baryons. So we add them up and write them as a single term. So this is the first term which corresponds to non-relativistic matter, dark matter plus baryons, which go as a cube $1/a^3$, then the radiation term which goes as $1/a^4$, then the cosmological constant term, which does not evolve with $a$. And if there are curvature we would have to add the curvature term but since there is no curvature we are done. Okay.

Now you see, there are various interesting effects one which we can look at. When $a$ goes to 0, that means the Universe is small and we are looking at very early times, you can see when if we put $a$ going to zero the $\Omega\Lambda$ will become negligible and at
sufficiently small a even this $\Omega$ matter will become negligible compared to the $\Omega$ radiation term. So at very early times, the radiation term will dominate. Take the other extreme, that when $a$ is large, then both of these terms will be negligible and it will be dominated by the cosmological constant. So at very early times the radiation dominates, at very late times the cosmological constant dominates, at intermediate times the non-relativistic matter dominates, the $\Omega$ matter.

[00:36:13] You can in fact calculate what are the values of this $a$, the scale factor, when the these various the transition from one domination to another domination occurs. For example at early times, the Universe is radiation dominated and then it becomes dominated by non-relativistic matter. So you can ask the question when does it happen? So that roughly happens when $\Omega$ matter and $\Omega$ radiation are equal so you can solve this equation and obtain a value of $a$, which will give you this equality. The value of $a$ inverse turns out to be somewhere around 3500.

[00:37:01] This epoch, which corresponds to $a$ equals to 1 by 3500 roughly is known as the epoch of matter radiation equality. Before this epoch the Universe was radiation dominated, after this epoch the Universe was matter dominated. Similarly, you can find the epoch where matter and cosmological constant were equal and that will give you a which is much closer to 1, much closer to present day that somewhere around 0.8 or so.

[00:37:32] This is a plot which shows the evolution of various density components as a function of scale factor. So the densities are normalised to the present day value of the critical density. Let us start with radiation, which is the blue curve. So this is the radiation curve and since the radiation goes as 1 by $a$ to the power 4 you have a curve like this. Remember both the axis are logarithmic, so any power law will show up as straight lines here.

[00:38:06] You can see at very early times when $a$ is small, the radiation dominates over all other forms of matter. But since this decreases very fast, by the time you come to today, which is let's say $a$ equals to 1, the the contribution of radiation have become negligible. It is below all other possible forms of energy or matter.

[00:38:31] If you take matter, by matter I mean the non-relativistic matter, that is this orange curve that goes as 1 by a cube. At early times, it is smaller than radiation. But at some point it becomes equal to radiation and then it overtakes radiation. So this would be the phase when the Universe is then matter dominated, you can see the orange curve is above all the other curves. So the Universe remains matter dominated for quite some time, till at some point the cosmological constant takes over.
So the cosmological constant is shown by the green curve. Since its constant this curve is basically horizontal. At early times, the cosmological constant is negligible compared to other forms of density. So if we were living in a Universe at this time, we wouldn't even know that the cosmological constant existed. But since it is constant and all the other forms are decreasing, all the other forms of densities are decreasing, at some point the cosmological constant takes over the radiation and it very recently it also takes over the matter and in the future the Universe will be completely dominated by the cosmological constant.

For completeness, I have also shown the evolution of the $H(a)$ square term, which is basically addition of these three terms which is shown by the red curve. So you can see the red curve at early times is on top of the radiation curve. Then it slowly transits over to the orange curve, which is the non-relativistic matter and here it has transited over to the cosmological constant curve. So this Hubble parameter at early it has this clearly these three phases, the radiation dominated, the non-relativistic matter dominated and the cosmological dominated phase.

Section 4.3.4
Difficulties with the standard model

Okay. Now this is our standard model of cosmology and it actually helps in explaining many many many observations. But there are some difficulties and before going into describing the standard model in more detail, let me just talk about some of the difficulties. So the difficulties which I have listed here and we will go through one by one are the horizon problem, the flatness problem and some difficulties related to the cosmological constant.

Of course, there is this another difficulty which we won't talk about much, is the fact that we don't have any understanding of this dark matter or this dark energy components. We don't know what they are. We don't know what this dark matter is, it is not detected in any laboratory or any terrestrial experiments. But at least it is something which has non-relativistic, it has zero pressure and probably it will come out from some extended theories of particle physics. Dark energy is completely out of hands because now we were talking about something which has negative pressure, extremely difficult to explain by any theory. So we really don't know what they are.

So let us come to the first problem, which is the horizon problem. As we have seen already, the age of the Universe is somewhere of the order of the inverse of the Hubble parameter. Now we can calculate the distance traveled by photons during this time. It is basically $c$ divided by $H$ inverse. So that's basically $c$ times the $t$. The actual distance
traveled will be slightly more if you do the full calculation, but for the moment let us just stick to this order of magnitudes and that's sufficient for our purpose.

Now if the photons can only travel a distance $c$ by $H$, $c$ by the Hubble parameter, then if there are galaxies beyond this distance, we will be we will not be in causal contact with those galaxies. So light from those galaxies would not have reached us because photons did not get time to travel that much. This is known as the particle horizon the fact that galaxies beyond the distance won't be in any kind of contact with us. Now we know how the Hubble parameter evolves with scale factor all with time. We just worked that out in the previous few slides.

In fact, the the horizon will go as a square during radiation-dominated era and it will go as $a$ to the 3 by 2 in the matter dominated era. This is something you can just see by looking at the expression for the Hubble parameter. So the horizon evolves as a square and $a$ to the 3 by 2 depending on which era you are in. The distance between two points, let's say two galaxies are any two points in a Universe, the that distance evolves as $a$, that is just the consequence of Hubble expansion. The distance is just increasing with the scale factor.

Now, let us plot various length scales as a function of $a$. So the red dashed curve shows the evolution of the horizon. Okay, the $cH$ inverse $a$. It is increasing as a square at early times, the radiation dominated phase and that it still increases but increases as $a$ to the 3 by 2, that will be the matter dominated phase. This vertical dashed line just shows you the matter radiation equality, Universe is transiting from the radiation dominated phase to the matter dominated phase.

Let us for the moment concentrate on this thick black line. Let us say you have two galaxies and you know what the distance between them is. That distance increases as $a$. Okay, so the distance at this $a$ is given by whatever this is and at early times, the distance is smaller just evolving as $a$. Now the point to note is that the proper distance between these two points goes as $a$, whereas, the Hubble radius or the horizon evolves faster than that. It involves either as a square or $a$ to the 3 by 2.

Now hence, as we go more and more in the past, so you go to smaller and smaller $a$, what will happen is that this $d$, the distance between these two points will become larger than the horizon. So this means that if we are thinking about two galaxies separated by this much distance, if you go to for example, let's say matter radiation equality here, their distance would be larger than the horizon. So till this point they were never in causal contact. Okay, you can see that distance is larger than the horizon. They came to causal contact first at this point, when they entered the horizon. After that they have been in causal contact, okay.
Now you can take any two galaxies and plot a line like this. For example, if you take two galaxies which are really really far away, like this one, these two galaxies this pair has not come to causal contact probably even now, because they are so far away that they haven't entered the horizon. These are two galaxies which are probably much much closely separated, these have come to closure contact the causal contact much earlier. They have entered the horizon much earlier and so on. Okay. However no matter which scale you take, particularly the cosmologically relevant scales, if you go early enough, it's clear that they will go out of the horizon and they won't be in casual contact at early times.

Now this is a problem because, if the galaxies which are separated by relevant cosmological scales were not in causal contact at early times, how did the Universe become so homogeneous? We look at the Universe all all around us. For example through the cosmic microwave background radiation or many other probes. We see them to be extremely homogeneous. The Universe is looks the same at all places. So if they were not in causal contact at early times, how did the Universe become so homogeneous? How did each point know what happens to the other point? Okay, it seems they all at ever at early times at least they all evolved kind of without knowing what the other guys are doing. So it's very difficult to explain homogeneity of the Universe, if we believe this picture, if we believe the fact or if we if we believe this calculation which shows that the distance between two galaxies or two points will become larger than the horizon at early times.

Okay. Now we can solve the horizon problem. If we want to solve the horizon problem, we have to introduce a phase of the Universe where the Hubble parameter or the horizon did not evolve faster than the the proper distance between galaxies. The best way to do that would be to take Universe where the Hubble parameter was simply a constant, like this, okay. So till in the last plot, we were talking about the radiation dominated phase and the matter dominated phase, but suppose there was a phase before the radiation dominated phase, where H was constant, so you make it constant like this. Then you see the distance between two galaxies would be less than the horizon at very early times.

So if you go to early enough times, you can see that all cosmologically relevant scales or distances, will become smaller than the horizon and then there is no problem. Okay. All cosmologically relevant quantities would have been in causal contact at very early times. Then in between of course they went out of causal contact, but that's not a problem. Because they were in causal contact at early times and the homogeneity was set in at that time. So we won't have any difficulty in explaining homogeneity if we have a phase where the Hubble parameter is almost constant.

If the Hubble parameter is constant you can again solve the Friedmann equation and you will be able to show that the scale factor will evolve exponentially. So what it...
Transcript

[00:50:11] Okay, so fine. So let's say that phase exists and we'll come back to it. Let us talk about another problem which is the flatness problem. Now this problem has to do with the evolution of the curvature density. So you take the $\Omega$ corresponding to curvature and workout it's evolution with respect to $a$, just from the expression from the of the Hubble parameter you should be able to show that the curvature evolves like this. So it is related to the curvature at present divided by the summation over all $\alpha$ components and the curvature term, okay.

[00:50:52] For the standard model of cosmology I can interpret this curvature as departure from $\Omega$ total minus 1. So if $\Omega$ total is equal to one curvature is zero, but if $\Omega$ total is not equal to 1 we what we get is the negative of curvature as $\Omega$ minus 1. So this would be given by a expression written here. Okay. So it's $\Omega$ total at to date minus 1 divided by all these terms.

[00:51:24] Now if I take the limit $a$ going to 0 what will happen is in the denominator, the radiation term will dominate so you will end up with a term which is just $\Omega r$ by a square and then you will take the $a$ to the numerator and if you take now the limit $a$ going to 0, what you find is that the left hand side becomes zero, no matter what you choose for $\Omega$ total today. So what it tells you is no matter what the $\Omega$ total is today, $\Omega$ total at very early times is very very close to 1. In other words, no matter what the curvature of the Universe today is, at early times the Universe was really really really flat, you can make it as flat as you want by taking going to earlier and early times.

[00:52:17] Just to take a specific example, suppose $\Omega$ total is point 5 today, of course we know it is not but for that moment let's assume that $\Omega$ total is point 5 today. Then at very early times, let us just for an example, you take $a$ to be 10 to the minus 8, quite early times in the Universe, what you have is the $\Omega$ total at this epoch will turn out to be very close to 1 and the departure is like 1 part in $10^{12}$ or $10^{13}$. So if I had written this $\Omega$ total $a$ at a equal to 10 to the minus 8 for a Universe, which had $\Omega$ equal to point 5 today, that would turn out to be all these nines and some digit. Okay, so you can see it is very very close to one and you can make it closer to 1 if you take a small smaller value of $a$.

[00:53:13] Now you say that suppose instead of starting with the Universe which has $\Omega$ total of this this many nines and a 4, you start it with something which is slightly different, is very close to 1 but the departure of 1 instead of it being 6 times 10 to the minus 30, let us say it's just 10 to the minus 12. So it is the difference between this and this so you can see
the difference is almost it is very small. However, if you start with this second case you would have ended up with a Universe which is very different, instead of point 5, you would have ended up with the $\Omega$ total of point 9.

[00:53:52] So that means the it tells you that the $\Omega$ total at early epochs must have to be very very very finely tuned towards unity. For example, even if you had taken this value you would have ended up with point 9. Okay, if $\Omega$ total is much closer to 1 you have to actually fine tune this value to much much more decimal points. Okay, so that looks a bit unnatural, why should a parameter be fine tuned to something to beyond 12 13? I mean, I don't know how many 20 30 decimal points. Okay. So this tuning of initial conditions which make physicists uncomfortable is called the flatness problem. So it it has to do with the fact that we have to tune the curvature parameter to a very high degree of accuracy to make the Universe look what it is.

[00:54:48] It turns out the flatness problem can also be solved by incorporating a rapidly expanding phase at early early times, which H almost constant. So to do that, what you can do is you can work out again the evolution of the curvature parameter or $\Omega$ minus 1 parameter.

[00:55:09] What you will find is that let us imagine that there there is a phase of this rapid expansion, it starts at ai and ends at af a initial and a final. What will happen is the $\Omega$ total at a final will be related to $\Omega$ total at a initial by an expression like this. So right hand side is a initial square by a final square. Now since the scale factor increases exponentially during this rapid expansion, af we expect it to be much much much larger than a initial. So what happens is the right hand side becomes 0. So in that case what happens is no matter what $\Omega$ total you start with, the $\Omega$ total at a final will simply go to 1 because the right hand side has gone to 0. So that then solves the flatness problem because you start with whatever Universe you want. Okay, you don't have to fine tune your $\Omega$ total.

[00:56:09] But then let the Universe expand rapidly. Okay, exponentially with Hubble parameter almost constant or constant. Then what will happen is the moment you end this expansion phase, at the end of this expansion phase, the $\Omega$ total will be almost 1, it will be all very very very close to 1. So what happens is you now don't have to fine tune by hand, you start with whatever you want, this expansion rapidly expansion phase will take care of this fine tuning and it will simply go to 1 because the right hand side has gone to 0. So that then solves the flatness problem because you start with whatever Universe you want. Okay, you don't have to fine tune your $\Omega$ total.

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quantum effects in the smooth Universe. These are the seeds which will grow and give rise to the large-scale structure we see around us today. So in that sense inflation is quite useful.

The expansion rate since it is exponential with Hubble parameter equal to constant this requires an energy component, which again is very similar to the cosmological constant. It needs a pressure very close to negative of energy density. This is usually achieved by a scalar field, which is slowly rolling in a flat potential $V(\phi)$, with this can be achieved by writing a Lagrangian of this form and then solving in the expanding Universe in the Friedmann–Robertson–Lemaître–Robertson–Walker metric.

To solve these problems, we think that the inflation should need to start around $10^{-36}$ seconds. So very very early and it should end around somewhere on $10^{-32}$ seconds. You can work out that by this time the scale factor of the size of the Universe would have increased by a factor of $10^{26}$. So it's a massive massively rapid expansion. The temperature of this Universe at that time would have would be corresponding to an energy which is something like $10^{15}$ GeV. So really really high energy. Okay. So this inflation is something which we don't understand how it is due to, its probably due to some scalar field, but it is very useful in solving the problems with a standard model of cosmology.

There are various difficulties related to the cosmological constant as well, which has to do with the late time acceleration. If you ask what is the cosmological constant, we don't know. Often people interpret this as the vacuum energy arising from quantum gravitational fluctuations. So in absence of everything else, this is something which contributes to the energy tensor of the Universe. Now if it is arising from quantum gravitational fluctuations, the natural energy scale for such an energy component would be the Planck energy, so you can then work out the vacuum energy corresponding to such a scale the Planck scale and it will turn out that the vacuum energy would correspond to something around $10^{93}$ grams per cc.

On the other hand, the energy density corresponding the cosmological constant is point 7 times the critical density. So that would turn out to be $10^{-29}$ grams per cc. So you can clearly see that the vacuum energy is much much larger than the cosmological constant which we observe and I mean it is larger by a huge amount. It's $10^{100}$ to the power some 100 something. Okay, so it's not like it's larger by a factor of few or anything which we can probably be able to deal with. But since it's several orders of magnitude smaller than the vacuum energy, it's very difficult to explain $\Lambda$ in terms of quantum gravitational fluctuation. So this is often known as the small cosmological constant problem. It's too small to be explained by quantum gravitational effects.
So again to explain the acceleration, people often uses a scalar field rolling in a potential. So the mathematics is often very similar to inflation, but it helps again in explaining the acceleration of the Universe, but there is a plethora of models for the scalar fields. They have they come with many names like quintessence, tachyons and so on and but we don't know which model is true because we don't have any fundamental understanding from any fundamental theory of particle physics or field theory and the data is not going to tell us that, which of these scalar field models are correct. It's probably only going to tell us the Ω parameter and what is the corresponding equation of state w for the cosmological constant.

So this again is a good place to stop. So today in this lecture, what we have done is we have talked about our Universe, what is it made of? How do we solve for various quantities relevant to our Universe? What are the constituents? And not only that, we also talked about some of the difficulties which exist for our Universe. So in the next lecture, we will talk a bit more about the standard cosmological models and what we know about it.