Chaos, catastrophe and mathematics

The effective way in which Newton’s methods handled various phenomena led scientists and mathematicians to the belief that the Newtonian framework can account for all diverse situations

History of mathematics tells us that in 1797, when a bank in London failed or not, the borrower will have to pay the bank a slightly more interest. The effective way in which Newton’s methods handled these and many other phenomena led scientists and mathematicians to the belief that the Newtonian framework can account for all diverse situations. Even lay persons outside science and maths came to feel that these inputs from Newton are sufficient to understand natural phenomena. A poet like Alexander Pope underscored his admiration for Newton in his poetic tribute when he wrote: “Nature and nature’s laws lay hid in night: God said let Newton be and there was light.”

But in the 20th century new branches of mathematics began to surface and it became clear that in many situations Newton did not have the last word. For example, Newtonian methods dealt with situations described above, which are of a “continuous” nature. But nature may have other types of situations to which Newtonian ideas cannot apply. Imagine the plight of a mountain climber who is scaling a steep hill. As he goes up, he suddenly discovers that the next stage involves negotiating a precipice. Or, think of a dogfight in which two dogs fight for a territory. After a nearly equal encounter, one dog decides that enough is enough and walks away leaving the territorial authority to its rival. Are there mathematical models, which help us understand better what is going on in such cases?

French mathematician René Thom showed how this can be done. There are phenomena characterised by abrupt changes which occur in a discontinuous mode. Such critical changes, Thom argued, may be characterised by the adjective “catastrophic”. These cannot be described by Newtonian methods but are amenable to the ideas and models of Thom’s catastrophe theory. One may apply the catastrophe theory to some of the celebrated battles like Waterloo or Panipat, or Sinhagad, where the events changed the outcome. A battle like Panipat left behind lasting changes and it is an interesting exercise to speculate what would have happened if the outcome of the battle had been different. In the campaign to win the fort of Sinhagad, the attacking leader Panaji was killed unexpectedly. But his uncle Shelar mama stepped in his place and revived the morale of the invading group to ensure victory. If at Panipat, the Marathas had won conclusively instead of losing disastrously, would subsequent history of the subcontinent have been different? And what would have been the political shape of Europe had Napoleon received the timely reinforcement at Waterloo? Relatively minor happenings in a battlefield could lead to a difference between victory and defeat.

In a similar vein one can mention another branch of mathematics called “chaos”. At first sight this name sounds strange. How can mathematics which, in the lay mind is associated with neatness and order, have any association with chaos? The answer is that the chaos itself may be of a kind that is generated by mathematics! The following example will illustrate this point. Take the ratio of circumference of a circle to its diameter, normally denoted by the Greek letter Pi. Now consider an example of generating a sequence of numbers selected by taking every third member in the decimal expansion of Pi. The decimal expansion proceeds as 3.141596... with the dots warning us that the expansion is endless. Thus, our sequence of every third member will begin 3, 1, 5, 6, etc. If you ask an expert what is the next number he will calculate and tell you. However, he cannot give a rule, which will tell us what is the number at a pre-specified order in this sequence. To anyone not knowing where these numbers are from, the sequence above would look chaotic, with no apparent order.

Like catastrophe, chaos talks of numbers or events with no apparent rule governing them, although some underlying mathematical method of their derivation exists. An exaggerated example of an event in meteorology is often given: a butterfly flapping its wings in Brazil develops weather conditions that lead to a tornado in Texas! In this case there may be an underlying reason but until we weather forecast develops as an exact science, the connection is qualitative at best.

Chaos, as a subject, as a part of mathematics tries to probe apparent chaotic conditions so as to find the underlying reason. From a steadily and gently flowing river to a Niagara type waterfall, where does chaos enter the scene and how do maths cope with the growing chaotic changes that we see? Such considerations show that beyond Newton there are territories for mathematicians to conquer. Catastrophe and chaos pose challenges to them and tempt them with glimpses of new vistas where even Newton feared to tread.

The writer, a renowned astrophysicist, is professor emeritus at Inter-University Centre for Astronomy and Astrophysics, Pune University Campus. He was Cambridge University’s Senior Wrangler in Maths in 1859.