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Abstract
In light of the recent discoveries of binary black hole events and one neutron star event by the advanced LIGO (aLIGO) and advanced Virgo (aVirgo) detectors, we propose a new astrophysical source, namely, the mini creation event (MCE) as a possible source of gravitational waves (GW) to be detected by advanced detectors. The MCE is at the heart of the quasi steady state cosmology (QSSC) and is not expected to occur in standard cosmology. Generically, the MCE is anisotropic and we assume a Bianchi Type I model for its description. We compute its signature waveform and assume masses, distances analogous to the events detected. The striking feature of the waveform associated with this model of the MCE is that it depends only on one amplitude parameter and thus allows for simpler data analysis. By matched filtering the signal we find that, for a broad range of model parameters, the signal to noise ratio of the randomly oriented MCE is sufficiently high for a confident detection by aLIGO and aVirgo. We therefore propose the MCE as a viable astrophysical source of GW. The detection or non-detection of such a source also hold implications for QSSC, namely, whether it is a viable cosmology or not.

Keywords: gravitational waves, mini creation event, LIGO–Virgo detection

(Some figures may appear in colour only in the online journal)

1. Introduction
The detection of gravitational waves in 2015 by the advanced LIGO (aLIGO) detectors has opened out a new avenue for the study of the cosmos. The detection itself indicates that there are cosmic sources which cannot be detected by electromagnetic radiation but whose gravitational signature is open for detection. Thus the first source to be detected by the Ligo detectors consisted of coalescing black holes [1] neither of which could be detected by the conventional
telescopes using electromagnetic radiation. Subsequently, several more binary black hole coalescences and one neutron star coalescence were detected having similar characteristics by aLIGO and advanced Virgo (aVirgo) [2–7].

The success of such detections opens up the possibility of investigating new gravitational waves (GW) sources which can be detected by the present detectors [2–7]. However, by ‘new’ sources, we mean objects emitting gravitational waves whose physical composition is significantly different from the first object detected by aLIGO. Already there are on-going efforts by the LIGO–Virgo collaboration to detect various types of astrophysical sources. These are the continuous wave sources—GW emitted by asymmetric neutron stars, burst sources from powerful transient phenomena, the stochastic background of GW emanating from independent, unresolved GW emitters etc [8]. The data analysis methods for the sources mentioned have progressed considerably and are continuously being improved. An additional and possible GW source that can be looked for is the MCE. A similar development occurred with detectors of electromagnetic waves when pulsars, quasars, x-ray sources, gamma ray bursts etc were detected. For example, radio astronomy provides a variety of sources like quasars, pulsars, large radio galaxies, radio stars etc, each source having its own distinguishing physical composition. Thus the possibility of detecting a new astrophysical source will always be of interest to advanced gravitational wave detectors.

The purpose of this paper is to propose a new source whose signature will be identified through match filtering methods used to extract signal from noise of a gravitational wave detector. The source proposed is the so-called mini-creation event (MCE) which was discussed in earlier papers [9–11]. The MCEs are pockets of matter creation distributed all over the universe, essentially replacing the big bang of standard cosmology. The large scale behaviour of the universe thus is determined by the MCEs. Indeed, as we shall demonstrate, the detection of gravitational waves in a certain pattern provides a test of cosmology predicting the existence of MCEs.

We summarize in the following section the salient features of the universe vis-a-vis the MCEs, the new cosmology being known as the quasi-steady state cosmology (QSSC in brief).

2. The minicreation events (MCEs)

2.1. The basic mechanism

The details of the QSSC may be found in a series of papers by its authors Hoyle et al [12–14]. The overall background to the work has been discussed in a book by the same authors [15]. Broad features of the theory relevant to the work of this paper may be summarized as follows.

(i) The field equations from which the QSSC models are derived are those obtained by [16] from an action at a distance formulation of Mach’s principle. Also, an important difference from standard relativity is that the cosmological constant is negative.

(ii) Like the steady state theory of [17] and [18], this theory gives nonsingular models with matter creation sustained by a negative energy scalar field $C$.

(iii) The $C$-field produces matter at a typical spacetime point provided the particles comprising of the created matter satisfy the equality,

$$m^2 = C_i C^i,$$  \hspace{1cm} (1)

where $m$ is the mass of created particle and $C_i$ stands for $\partial C/\partial x^i$, $x^i (i = 0, 1, 2, 3)$ being the time space coordinates. Theory suggests that the created matter is in the form of Planck particles of mass $(\hbar c/G)^{1/2} \equiv m_P$. For details see [12].
(iv) In general this condition is not satisfied since the density of the C-field, $C_iC_i$, is very low. But near a collapsed object of mass $M$, the value of $C_iC_i$ is raised. If $r$ is the radius of the collapsed object, then $C_iC_i$ is raised by a factor,

$$\gamma = \left(1 - \frac{2GM}{c^2r}\right)^{-1/2}.$$  \hspace{1cm} (2)

Thus the creation condition is satisfied provided $r$ is small enough and near $2GM/c^2$.

(v) A collapsing massive object in this theory is not headed for a black hole state followed by singularity but bounces at a finite radius $r_{\text{min}}$ because of the repulsive effect of the C-field owing to its negative energy. Thus provided,

$$m^2 \left(1 - \frac{2GM}{c^2r_{\text{min}}}\right)^{-1/2} > m_P^2 \equiv \frac{\hbar c}{G}$$ \hspace{1cm} (3)

we have creation of matter near highly compact massive objects.

The equation (3) indicates how difficult it is to create new particles near a collapsed massive object. The asymptotic value (away from the collapsed object) of $C_iC_i$ decides if creation takes place. If the inequality of equation (3) holds, more particles will be created. To what extent they will be created will be decided by how strong is the inequality of equation (3). A detailed quantitative theory will be needed to solve this problem.

We also note that if the inequality of equation (3) does not take place, no creation is triggered off. This is an important issue that also needs further consideration which may form the subject of a future paper.

Here we assume that there are some massive objects which raise the level of the C-field such that the above inequality is satisfied for typical particle mass $m$. Such massive objects act as centres of explosive creation because any new particle created is pushed outwards by the negative energy C-field owing to the repulsive effect produced by its negative energy.

(vi) Thus instead of a massive object becoming a black hole, it becomes a centre of creation. This is called a minicreation event (MCE). The MCE is very similar to a black hole so far as its contracting phase is concerned. It is different from a black hole in that it bounces when close to what would have been the horizon of a black hole. An MCE may also be referred to as a ‘minibang’. Although its outward behaviour may be like that of big bang it differs from the latter because it does not have a singular beginning, and its explosive creation does not violate the fundamental law of matter-energy conservation.

(vii) The universe as a whole responds to these MCEs happening all over it. It can be shown that the universe has a long term steady expansion at an exponential rate along with a short term behaviour of oscillatory nature. A simplified scale function of the universe is approximated by,

$$S = \exp(t/P) \{1 + \xi \cos (2\pi t/Q)\}. \hspace{1cm} (4)$$

The early work by Sachs et al [19] shows the cosmological solutions of the field equations for homogeneous and isotropic world models analogous to those of standard cosmology. The typical solution is specified by the parameters $P$, $Q$, $\lambda$ and $\xi$. The longer time scale characterises long term expansion whereas the shorter time scale $Q$ denotes oscillations. The oscillations are non-singular as indicated by the inequality $0 < \xi < 1$ of the parameter $\xi$. 

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Just as in standard cosmology, the model parameters are fixed by observation, so do the parameters $P$, $Q$, $\xi$ and $\lambda$ in the above models. The cosmological constant is negative: but this does not cause any problem as we will shortly see.

The $C$-field, however, plays an important role in the dynamics of the model. For example, when there is no creation of matter, the energy tensor of the $C$-field has zero divergence and it leads to the simple result $\dot{C} \propto 1/S^2$ for the above models, and hence the $C$-field energy density behaving as $\propto S^{-4}$.

The short-term oscillations reflect the condition whereby the MCE population pulses up and down. At the minimum of $S(t)$ the $C$-field is strong and more massive objects satisfy the inequality (3). This results in local expansion increasing. As $S(t)$ increases $C^2C_t$ decreases and the massive objects drop out of the creative mode. This ultimately gives rise to a contraction of the universe because of negative $\lambda$. Also, as stated before, because $|\xi|$ is less than unity $S(t)$ never becomes zero. Typically, we may take $P \approx 10^3$ Gyr and $Q \approx 50$ Gyr. Also, we take $\xi = 0.8$ to fix ideas.

The QSSC does not have a singularity, nor does it have a high temperature phase. The maximum redshift observed in the cosmology is not expected to go beyond the range of 10–20. Nevertheless the cosmology explains the observed microwave background radiation (MBR) as well as the abundances of light nuclei, vide [14] for MBR and [12] for nucleosynthesis. The newly created Planck particle decays in a time scale of $\sim 10^{-43}$ s. The decay of Planck particle into baryons, leptons, etc leads to local high temperature. The ‘Planck fireball’ evolves and leads to the observed light nuclei [12]. The oscillatory cycle which lasts for $\sim 50$ Gyr is sufficient for ordinary (Sun-like) stars to be born, evolve and then decay or explode leaving behind dark remnants. It is suggested that these remnants contribute the observed dark matter. What happens to the light emitted by stars in a cycle? This is thermalized and is seen as MBR.

The MBR in the QSSC is produced from the radiation of burnt out stars of preceding cycles. The stellar activity in the present cycle allows us to estimate the energy density associated with MBR. Thus calculations show that energy density at present epoch works out close to black body radiation of $\sim 2.7$ K. Thus, provided a thermalizing agent is present, one not only understands why this background radiation is present but also its intensity.

The thermalizing agent is metallic whiskers. The metals are created in stellar evolution and they are ejected by supernovae. Laboratory data show [20, 21] that metallic vapours cool down to condense as whiskers of typical lengths of up to $\sim 1$ mm. These are of cross section $\sim 0.01 \mu m$ and are able to absorb and reemit starlight so as to achieve thermalization.

The radiation so produced has been studied within the framework of QSSC. For a detailed study see [15]. We may mention that the QSSC had predicted the ‘Boomerang peak’ in the MBR power spectrum. Although a different physical origin may produce different sources of inhomogeneities, it is possible to compare the above concept with the standard interpretation. In particular, the role of intergalactic dust cannot be ignored in the QSSC.

Finally we have a second look at the Type Ia supernovae and the claim of accelerating universe. In the QSSC the intergalactic dust dims the distant supernovae and can thus provide an alternative interpretation of their dimming. (See [22, 23] for details.)

This is a brief survey of the QSSC to which we now add gravitational waves as providing additional checks. As was shown in the previous paper [11] that apart from discrete source observations studies of continuum gravitational wave background can in principle be compared with the post-inflation background produced in standard cosmology. Thus, here is a test of cosmological background produced by sources. Such a test may be possible at a future date.
While a comparison of backgrounds of gravitational waves is a possible way of distinguishing between different cosmological models, a more practical method of checking cosmological predictions is to try and detect an MCE. For an MCE is required by the QSSC whereas it is not expected to occur in standard cosmology. The recent detections of discrete gravitational wave emitters mentioned before of black hole binaries, however, suggest that at the current level of detection technology, looking for specific sources is likely to be a more fruitful approach.

Gravitational wave events were observed by the aLIGO detectors in their first and second run O1 and O2 and sometime during the second run advanced Virgo joined in. Several confirmed detections of binary black hole coalescences were made and most interestingly on 17 August 2017 a binary neutron star coalescence was observed [6]. This GW event was followed up by the observation of electromagnetic counterparts in several bands from γ-rays to radio [24]. So far the events observed are those of mergers of binary black holes and one neutron star merger whose signature waveforms have been computed analytically and numerically. Therefore, one knows what one is looking for and uses the match filtering methods [25] to extract the signals from the noise. It is possible that the data contain signals from other astrophysical sources and if so one should endeavour to detect and identify such signals. As mentioned earlier, efforts in the LIGO–Virgo collaboration are ongoing to look for other types of astrophysical sources which have the potential of generating detectable GW by current detectors. The astrophysical source we propose here is the MCE. However, in order to detect such a source one needs to know the signature of the signal by computing the waveform. In this calculation we compute the GW waveform based on the model of the MCE proposed in [9].

2.2. Gravitational radiation from an arbitrarily oriented anisotropic MCE

Although a typical MCE is nonsingular in origin, being made of ejected newly created matter, the model assumed here will be approximated by a triaxial ellipsoid expanding anisotropically in all directions. While in general relativity such a solution is singular, in our modified theory it will have arisen by a bounce at a small size. Thus replacing this minimum size by zero will not produce much error.

Because of the anisotropy we expect the MCE to emit gravitational waves. The expansion is described via a Bianchi Type I model whose metric is given by:

\[ ds^2 = c^2 dt^2 - X^2(t) dx^2 - Y^2(t) dy^2 - Z^2(t) dz^2, \]  

(5)

where \((x, y, z)\) are the comoving coordinates and \(t\) is the proper time of a dust particle moving outwards. Solving Einstein’s equations for dust dominated systems we have:

\[ X(t) = S(t)[F(t)]^{2\sin \gamma}, \]
\[ Y(t) = S(t)[F(t)]^{2\sin(\gamma + \frac{2\pi}{3})}, \]
\[ Z(t) = S(t)[F(t)]^{2\sin(\gamma + \frac{4\pi}{3})}, \]  

(6)

where,

\[ F(t) = \left(\frac{GM}{S(t)}\right)^{1/3}, \quad S^3(t) = X(t)Y(t)Z(t). \]  

(7)

The anisotropy is related to the parameter \(\gamma\) which varies between \(-\pi/6\) to \(\pi/2\). The average scale factor \(S\) is related to the mass \(M\) by:
\[ S^3(t) = \frac{9}{2} GM(t + \Sigma), \quad \Sigma = \text{const.} \]  

Such an expanding object has time varying quadrupole moment and will emit gravitational waves. Again, we stress that the presence of singularity at \( t = 0 \) will not alter the conclusion in any significant way.

The source frame of the MCE is denoted by \((x, y, z)\) and the MCE has principal axes as the coordinate axes. Thus the quadrupole tensor is diagonal because of the inherent symmetry assumed in the model and can be easily computed—there are no off-diagonal terms. It is given by:

\[ I_{xx} = \frac{1}{5} M X^2(t), \quad I_{yy} = \frac{1}{5} M Y^2(t), \quad I_{zz} = \frac{1}{5} M Z^2(t). \]  

We consider the situation when \( t \gg \Sigma \). Then the following simplifications occur. We have:

\[ S(t) = \left( \frac{9}{2} GM \right)^{1/3} t^{2/3}, \quad F(t) = \left( \frac{2}{9} \right)^{1/3}. \]  

Also,

\[ X(t) = (GM)^{1/3} t^{1/3} \left( \frac{2}{9} \right)^{1/3} \sin \gamma - \frac{2}{3}, \]  

with \( Y(t) \) and \( Z(t) \) described by similar expressions where \( \gamma \) is replaced by \( \gamma + 2\pi/3 \) and \( \gamma + 4\pi/3 \) respectively. Note we could have switched to labelling the \((x, y, z)\) axes as \((x_1, x_2, x_3)\) but we do not do this in anticipation of what follows. We need to transform from the source frame to the wave frame in order to get the wave amplitudes and the usual convention in the literature is to use \((X, Y, Z)\) for the source frame and \((x, y, z)\) for the wave or radiation frame and so we follow this notation as in [26].

From these expressions the quadrupole tensor can be readily computed. From equations (9) and (11), its components are given by:

\[ I_{xx} = \frac{1}{5} M (GM)^{2/3} t^{1/3} \left( \frac{2}{9} \right)^{1/3} \sin \gamma - \frac{4}{3}, \]

with similar expressions for \( I_{yy} \) and \( I_{zz} \), in which \( \gamma \) is replaced by \( \gamma + 2\pi/3 \) and \( \gamma + 4\pi/3 \) respectively.

The GW strain amplitudes are proportional to the second time derivatives of the quadrupole tensor evaluated at the retarded time \( t - R/c \), where \( R \) is the distance from the observer to the MCE. Explicitly,

\[ h^{TT}_{ik}(R, t) = \frac{2G}{c^4} \frac{1}{R} \left[ \dot{I}_{ik}(t - R/c) \right]^{TT}. \]  

The superscript \( TT \) refers to the transverse-traceless gauge. With this as our goal we define the basic GW strain amplitudes \( h_1, h_2, h_3 \) which incorporate the second time derivative of the quadrupole tensor at the retarded time. These strain amplitudes appear in the final GW amplitudes in the radiation frame. We define:

\[ h_k(\gamma) = \frac{A}{R} \left( t - \frac{R}{c} \right)^{-2/3} \left( \frac{2}{9} \right)^{1/3} \sin \left( \gamma + (k-1)\frac{2\pi}{3} \right), \]
where \( k = 1, 2, 3 \) and,
\[
A = \frac{4}{5} \left( \frac{2}{9} \right)^{1/3} \left( \frac{GM}{c^4} \right)^{5/3}, \tag{15}
\]
is a constant amplitude. Note that these basic strain amplitudes \( h_k \) depend on the parameter \( \gamma \).

We next compute the two polarisation GW strain amplitudes in the wave or radiation frame. The source frame is denoted by the \((x, y, z)\) frame. The coordinate axes are also chosen to be the principal axes of the MCE. However, in general, the MCE can have arbitrary orientation and therefore the source frame can be arbitrarily oriented with respect to the observer. We therefore need to compute the two GW polarisation amplitudes denoted by \( h_+ \) and \( h_\times \) in the wave frame. The wave frame is denoted by \((X, Y, Z)\). We therefore rotate the source frame to the wave frame by the Euler angles \( \alpha, \iota, \beta \) using the Goldstein convention—first rotation by angle \( \alpha \) about the \( z \)-axis, then second rotation about the line of nodes (new \( x \)-axis) by angle \( \iota \) and the final rotation about the new \( z \)-axis by angle \( \beta \). However, if we just need to point the new \( z \)-axis along the line of sight (negative \( Z \) axis), then only the first two rotations are necessary, namely, by angles \( \alpha \) and \( \iota \). But then the orientation of the \( X - Y \) axes will be determined by the orientation of the source—this may be sufficient for certain purposes, but in general is not desirable. If we fix the wave frame then another rotation by the angle \( \beta \) is necessary to make the rotated \((x, y)\) axes coincide with the \((X, Y)\) axes. Here we give the GW amplitudes for both situations.

The transverse and traceless components of the metric perturbation in the wave frame give the two GW polarisation amplitudes. We first consider only the two rotations by angles \( \alpha \) and \( \iota \). We call these amplitudes \( h_{+0} \) and \( h_{\times0} \). They are given by:
\[
\begin{align*}
 h_{+0} &= \frac{1}{2} (1 + \cos^2 \iota) \cos 2\alpha \, h_1(\gamma) + \sin^2 \iota \, h_2(\gamma) \\
 h_{\times0} &= \cos \iota \, \sin 2\alpha \, h_1(\gamma),
\end{align*}
\tag{16}
\]
where,
\[
 h_1(\gamma) = \frac{1}{2} (h_1 - h_2), \quad h_2(\gamma) = \frac{1}{4} (h_1 + h_2 - 2h_3), \tag{17}
\]
where \( h_1, h_2, h_3 \) have already been defined in equation (14).

Including also the third rotation by the angle \( \beta \), we can write the final GW strain amplitudes \( h_+ \) and \( h_\times \) in the wave frame in terms of the amplitudes \( h_{+0} \) and \( h_{\times0} \) just by using the tensor transformation law:
\[
\begin{align*}
 h_+ &= h_{+0} \cos 2\beta - h_{\times0} \sin 2\beta, \\
 h_\times &= h_{+0} \sin 2\beta + h_{\times0} \cos 2\beta. \tag{18}
\end{align*}
\]
We can now explicitly write these amplitudes in terms of \( h_1(\gamma) \) and \( h_2(\gamma) \).
\[
\begin{align*}
 h_+ &= \left[ \frac{1}{2} (1 + \cos^2 \iota) \cos 2\alpha \cos 2\beta - \cos \iota \sin 2\alpha \sin 2\beta \right] h_1(\gamma) \\
 &\quad + \sin^2 \iota \cos 2\beta \, h_2(\gamma) \\
 h_\times &= \left[ \frac{1}{2} (1 + \cos^2 \iota) \cos 2\alpha \sin 2\beta + \cos \iota \sin 2\alpha \cos 2\beta \right] h_1(\gamma) \\
 &\quad + \sin^2 \iota \sin 2\beta \, h_2(\gamma). \tag{19}
\end{align*}
\]
3. Astrophysical MCE and their detection by advanced detectors

In order to fix ideas, let us first consider the simple case of an MCE whose $z$ axis points along the line of sight and also the $(x,y)$ axes coincide with the $(X,Y)$ wave axis so that $\alpha = \beta = \iota = 0$. This implies that only $h_+ \neq 0$, that is, $h_\times = 0$ and that $h_+ = h_1(\gamma)$. We therefore take the GW strain $h_{xy}$ to be just $h = h_+ \equiv h_1(\gamma)$. We then find,

$$h_{xy} = \frac{1}{2}(h_1 - h_2) = \frac{A}{R} \tau^{-2/3} \eta_{xy},$$

(20)

where the amplitude $A$ has been defined in equation (15), $\tau = t - R/c$ is the retarded time and the anisotropy parameter $\eta_{xy}$ is given by,

$$\eta_{xy}(\gamma) = \frac{1}{2} \left[ \left( \frac{2}{9} \right)^{4/3} \sin \gamma - \left( \frac{2}{9} \right)^{4/3} \sin(\gamma + \frac{2\pi}{3}) \right].$$

(21)

Note that the $\eta_{xy}$ defined here is half that of defined in [9]. The anisotropy parameter takes values of the order of unity over the range of permissible $\gamma$. A plot of $\eta_{xy}$ versus $\gamma$ has been shown in figure 1 as the dashed curve. We now evaluate $h_{xy}$ for typical astrophysical parameters. We take these parameters to be about those of the recent black hole binary events discovered. For example, the last event discovered GW170104 in O2 has a mass of about $50 M_\odot$ and was at an estimated luminosity distance of 880 Mpc. We therefore take the mass of the typical MCE to be $\sim 50 M_\odot$ and to be at a distance of $\sim 1$ Gpc. These values give,

$$h_{xy} \sim 4.57 \times 10^{-24} \left( \frac{M}{50 M_\odot} \right)^{5/3} \left( \frac{R}{\text{Gpc}} \right)^{-1} \tau^{-2/3} \eta_{xy}.$$

(22)

This is well within the range of the advanced detectors. However, this is not the largest value that the GW strain can achieve for the MCE considered. In fact we find that this MCE model is more anisotropic in the $(x,z)$ and $(y,z)$ axes and thus will produce larger GW amplitudes orthogonal to these axes. In fact, the fair course to take is to average the GW strain over all orientations. Since we have no apriori knowledge, we will assume a uniform distribution of orientations and average accordingly over the angles $\alpha, \beta$ and $\iota$. The result is that we obtain an average or root mean square (rms) value of the GW strain which we denote by $h_{\text{rms}}$ which in turn can be expressed in terms of the anisotropy parameter $\eta_{\text{rms}}$. We then have:

$$\eta_{\text{rms}}(\gamma) = \frac{2}{\sqrt{15}} \left[ \eta_{xy}^2 + \eta_{xz}^2 + \eta_{yz}^2 \right]^{1/4}.$$

(23)

In figure 1, we have plotted both $\eta_{xy}(\gamma)$ (dashed curve) and $\eta_{\text{rms}}(\gamma)$ (solid curve).

In terms of $h_{\text{rms}}$ the GW strain is given by,

$$h_{\text{rms}}(\tau) = \frac{A}{R} h_{\text{rms}} \tau^{-2/3},$$

$$\approx 4.57 \times 10^{-24} \left( \frac{M}{50 M_\odot} \right)^{5/3} \left( \frac{R}{\text{Gpc}} \right)^{-1} \eta_{\text{rms}} \times \tau^{-2/3}.$$  

(24)

From figure 1, we see that $h_{\text{rms}}$ can go up to almost 2.6 and therefore such a source should be observable by the advanced detectors. In order to check this we must compute the signal to noise ratio (SNR) of such an arbitrarily oriented event. For this we need the GW strain $h_{\text{rms}}$ in
the Fourier domain. Taking the Fourier transform of $h_{\text{rms}}(\tau)$ from equation (24) and then taking its absolute value for $f > 0$ (only this is needed to compute the SNR), we obtain:

$$|\tilde{h}_{\text{rms}}(f)| \simeq 6.63 \times 10^{-24} \left( \frac{M}{50 M_\odot} \right)^{5/3} \left( \frac{R}{\text{Gpc}} \right)^{-1} \eta_{\text{rms}} f^{-1/3}. \quad (25)$$

We now go on to calculate the SNR. We use the one sided noise power spectral density (PSD) of aLIGO corresponding to zero detuned high power. This is normally given numerically, but for our purpose the analytic fit given in [27] suffices. The results will at most differ by a percent or so which is acceptable under the circumstances. The analytical fit to the noise PSD is given by:

$$S_h(f) = 10^{-48} (0.0152 x^{-4} + 0.2935 x^{9/4} + 2.7951 x^{3/2} - 6.5080 x^{3/4} + 17.7622), \quad (26)$$

where $x = f/245.4$. The lower frequency cut-off is assumed to be 20 Hz. The SNR which we denote by $\rho$ of the rms signal is then given by:

$$\rho = 2 \left[ \int_{f_{\text{low}}}^{f_{\text{up}}} \frac{df}{df} \left| \tilde{h}_{\text{rms}}(f) \right|^2 \frac{S_h(f)}{S_h(f)} \right]^{1/2}. \quad (27)$$

Taking the lower cut-off $f_{\text{low}}$ to be 20 Hz and the upper cut-off $f_{\text{up}}$ to be 1 kHz, we then obtain:

$$\rho(\gamma) \sim 14.36 \times \left( \frac{M}{50 M_\odot} \right)^{5/3} \left( \frac{R}{\text{Gpc}} \right)^{-1} \eta_{\text{rms}}(\gamma). \quad (28)$$

In this frequency band most of the SNR is accumulated and yields fairly accurate results. Since a typical value of $\eta_{\text{rms}} \sim 2$, we typically have $\rho \sim 30$. When $\gamma \sim \pi/6$, $\eta_{\text{rms}}$ attains more or less its maximum value $\eta_{\text{rms}} \sim 2.58$ for which we have $\rho \sim 37$. 

**Figure 1.** The dashed curve shows $\eta_{xy}$ corresponding to the $(x, y)$ axes and the continuous curve shows $\eta_{\text{rms}}$ uniformly averaged over all orientations as a function of $\gamma$ where $-\pi/6 \leq \gamma \leq \pi/2$. Since both $\eta_{xy}$ and $\eta_{\text{rms}}$ are shown in the same plot we have labelled the vertical axis by just $\eta$. 

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However, the SNR we have calculated is for a detector orientation most favourable to the source direction. But in general this will not be the case and the SNR will be reduced because of angular factors. For a source direction described by the angles $\theta, \phi$ in the frame of the detector, the signal $h(t)$ is given by:

$$h(t) = h_+(t)F_+(\theta, \phi) + h_\times F_\times(\theta, \phi),$$

(29)

where $F_+(\theta, \phi), F_\times(\theta, \phi)$ are the antenna pattern functions (see [26]) given in terms of the direction angles $\theta, \phi$. Since we do not know from which direction the signal will arrive, and given our lack of any apriori knowledge, we may assume a uniform distribution of sources over the sky directions and then average over the sky directions. A simple calculation shows that the reduction factor is $2/5$ in the signal amplitude and therefore also in the SNR. For the typical value of $\eta_{\text{rms}} \sim 2$, the sky averaged SNR will be $\sim 12$ and for maximum value of $\eta_{\text{rms}}$, the sky averaged SNR $\sim 15$. Such high SNRs for black hole binaries generally imply confident detection. We may expect this to be the case here also. However, a detailed analysis, which we do not perform here, involving real data is necessary in order to confirm this.

The recent detections of black holes suggests that there exists a large population of merging binary black holes in the universe which would produce a GW stochastic background. Similarly, the superposition of unresolvable MCEs would produce a GW stochastic background. This question has been addressed in an earlier paper [11]. There a uniform distribution of MCE in space is assumed and their number density has been computed in terms of the MCE and QSSC parameters. In fact, the observed event rate of the MCE will constrain the MCE and the QSSC model parameters. For typical parameters of the QSSC model and the MCE, the GW background was found to be $\Omega_{\text{GW}}(f) \sim 10^{-12}$ at $f \sim 10$ Hz for an MCE which lasts 1000 s. For more favourable parameters of the MCE, this quantity could be as high as $\sim 5 \times 10^{-11}$ which would be in the detection ballpark of the Einstein telescope [8].

4. Future outlook

The first GW observation and others following it demonstrated the existence of not only of black holes in binary systems but also of black holes themselves. While the early detection of such sources has enhanced astrophysics, it also raises the interesting question as to whether there exist other types of sources of different composition. In the early days of astronomy a steadily shining star was commonly known; but later discovery of supernovae, nebulae, etc enriched the subject. In the same way we expect the subject of gravitational waves will get richer with the findings of different types of sources. To this end, in this article, we make a case for the MCE as a possible astrophysical source for GW astronomy. In the current situation of the data containing non-Gaussian, non-stationary noise, more than a single detector with uncorrelated noise is required to make a detection with acceptable confidence. Moreover, using more than one detector increases the SNR; for two detectors of similar sensitivity operating simultaneously, the SNR will increase by a factor of $\sqrt{2}$. In the estimates of the SNR that we have obtained here, we have assumed a single aLIGO detector operating at design sensitivity. If one were to consider making a detection of the MCE from current detector data or the data that we expect in the near future, procedure analogous to the one for detecting binary black holes will have to be followed. One would have to first match filter the data, look for coincidences in the time of arrival and most importantly estimate the noise background. The noise background can be estimated by performing time slides, where one time slides the filtered output of one detector relative to the other with durations comfortably larger than the GW travel time between the detectors—this is $\sim 10$ milliseconds for the aLIGO detectors. We
would like to mention here that, in case of the MCE, the analysis would be simplified in one important way as compared to the one used in the detected binary black hole events—there is only one template for the simple model of the MCE that we have considered here—the signal has just one amplitude parameter, as compared to the hundreds of thousands of templates required for binary black hole coalescences where the waveforms depend on several parameters. Further, in future if more detectors come on line, for example, the Japanese KAGRA detector, or Ligo-India then the chances of detecting such events will greatly increase. GW astronomy is set to herald a new era in fundamental physics, cosmology and astrophysics.

Finally, the MCEs proposed here provide an interesting cosmological test. As they are organically related to the QSSC, their detection or otherwise will tell us if the QSSC is a viable cosmology or not.

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References


