Bulk Motion of Ultrarelativistic Conical Blazar Jets

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ABSTRACT
Allowing for the conical shape of ultrarelativistic blazar jets with opening angles of a few degrees on parsec-scales we show that their bulk Lorentz factors and viewing angles can be much larger than the values usually inferred by combining their flux variability and proper motion measurements. This is in accord with our earlier finding that such ultrarelativistic (Lorentz factor, $\Gamma \approx 30$) conical jets can reconcile the relatively slow apparent motions of VLBI knots in TeV blazars with the extremely fast flows implied by their rapid $\gamma$-ray variability. This jet geometry also implies that de-projected jet opening angles will typically be significantly underestimated from VLBI measurements. In addition, de-projected jet lengths will be considerably overestimated if high Lorentz factors and significant opening angles are not taken into account.

Key words: Blazars: general — galaxies: active — galaxies: jets — galaxies: nuclei — quasars: general — radio continuum: galaxies

1 INTRODUCTION
Although there is a consensus about the synchrotron origin of the radio emission from blazar jets, the values of the bulk Lorentz factor, $\Gamma$, and the misalignment angle from the line-of-sight, $\theta$, remain two key unknowns for any particular jet. Very Long Baseline Interferometry (VLBI) measurements of apparent motion of the parsec-scale radio knots have often been employed to constrain a combination of $\Gamma$ and $\theta$ (e.g., Vermeulen & Cohen 1994; Jorstad et al. 2001a). The degeneracy is broken by combining these data with additional observations, such as flux variability, or high-energy photons arising from (presumably) the synchrotron self-Compton (SSC) mechanism, when such data are available. Failure this availability, the jet parameters are frequently estimated by simply setting $\theta$ equal to its most probable single value (i.e., $\theta = 1/\Gamma$) (e.g., Vermeulen & Cohen 1994; Chiaberge et al. 2000).

In all these studies, it has been customary to assume a single value for the dominant emitting region, and we shall also do so here, for illustrative purposes. We note that many of the complications added by the possibility of a range in $\Gamma$s in a given jet have been discussed (e.g., Lind & Blandford 1985; Vermeulen & Cohen 1994). It has also been customary to assume (often implicitly) a narrow cylindrical geometry for the jet. As recently emphasized by us, this is a valid assumption only provided the opening angle of the relativistic jet, $\omega$, is much smaller than its beaming angle, $\psi \sim \Gamma^{-1}$,

(Gopal-Krishna, Dhurde & Wiita 2004, hereafter referred to as GDW).

It is important to note that on the sub-parsec scale, jets are probably still in the collimation regime, so $\omega$ is likely to be several degrees, as found, e.g., for the M87 jet (Junor, Biretta & Livio 1999). The evidence for this conical jet hypothesis continues to grow. Recently, Tavecchio et al. (2004) have provided evidence that the only two blazar jets for which their study could be carried out remain conical from sub-pc to $\sim 100$ kpc scales. Jorstad et al. (2005) have been able to measure projected opening angles for 15 blazar, quasar and radio galaxy jets using multi-epoch VLBA images taken at 43 GHz. They find projected half-opening angles between $2.4^\circ$ and $37^\circ$ and then estimate true full-opening angles, $\omega$, between $0.2^\circ$ and $7.6^\circ$, with a mean of about $2^\circ$.

As a result of this finite opening angle, for small inclination angles, the Doppler boosting of an ultra-relativistic jet ($\Gamma \gg 10$) as well as the apparent proper motions, can greatly vary across the jet’s cross-section, even when the relevant Lorentz factor for each knot is constant. It can then be important to carry out an integration of various quantities across the jet cross section. We recently showed that for an ultra-relativistic jet, such a refinement can often result in a drastic reduction of the apparent superluminal motion, compared to the canonical estimates (GDW).

We thus argued that it is possible to reconcile even very
large values of $\Gamma$ (approaching 100), which are favored by many models of TeV $\gamma$-ray emission (e.g., Mastichiadis & Kirk 1997; Krawczynski et al. 2001) with the rarely observed presence of apparently superluminal motions in TeV blazar jets (e.g., Piner & Edwards 2004; Giroletti et al. 2004), without invoking very large velocity gradients across the jet, very rapid deceleration, or extremely unlikely viewing angles (GDW). While the first of those alternatives, particularly the idea of a spine-sheath geometry on parsec scales (e.g., Chiaberge et al. 2000; Ghisellini et al. 2004) retains some attractions, the result that blazar jets appear to retain roughly the same Lorentz factor all the way out to multi-kpc scales when reasonable estimates can be made (Tavecchio et al. 2004; Jorstad & Marscher 2004) militates against the generality of the deceleration picture of Georganopoulos & Kazanas (2003) and the extremely small viewing angle possibility is statistically unlikely (Piner & Edwards 2004).

The possibility is statistically unlikely (Piner & Edwards 2004). The production of ultrarelativistic velocities through a "hydrodynamical booster", which is only possible in relativistic flows has recently been shown to be viable by Aloy & Rezolla (2006).

In this communication, we present a quantitative application of our approach to the procedure that is usually followed to infer $\Gamma$, by combining the apparent (often superluminal) speed, $v_{app}$, of VLBI components with the estimated value of the bulk Doppler factor, $\delta$, of the jet whose axis makes an angle $\theta$ from the line-of-sight. We find that the inferred values of both $\Gamma$ and $\theta$ are often substantially smaller than their actual values. We also show that the deprojected opening angle of the parsec-scale ultrarelativistic jet frequently is considerably smaller than the physical opening angle, and thus in accord with recent estimates (e.g., Jorstad et al. 2005).

2 CONICAL JET MODEL

The standard expression for the Doppler factor is

$$\delta = [\Gamma(1 - \beta \cos \theta)]^{-1},$$

with $\nu = \beta c$ and $\Gamma = (1 - \beta^2)^{-1/2}$. The corresponding expression for the apparent speed is

$$\beta_{app} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}.$$  \hfill (2)

There are several ways of estimating $\delta$ for a blob emitting incoherent synchrotron radiation. For instance, when the angular size of the blob is known from VLBI, one can compute (i) the inverse Compton Doppler factor, $\delta_{ic}$, using x-ray emission assumed to be of SSC origin (e.g., Marscher 1987; Ghisellini et al. 1993), or (ii) the equipartition Doppler factor, $\delta_{eq}$, using the radio spectral turnover (Readhead 1994; Guerra & Daly 1997; also, Singal & Gopal-Krishna 1985). If VLBI data at multiple epochs for multiple components is available, a $\delta_{app}$ can be found by assuming that the highest $\beta_{app}$ defines the minimum Lorentz factor, so $\delta_{app} \sim \beta_{app}^{max}$ (e.g., Jorstad et al. 2005).

Alternatively, $\delta$ can be estimated from radio observations of flux variability associated with a new VLBI component ("knot"), by adopting some maximum physically attainable value for the intrinsic brightness temperature, $T_{max}$ (e.g., Valtaoja et al. 1999). This $T_{max}$ could be set either by the equipartition condition ($\sim 5 \times 10^{10}$ K, Readhead 1994; Singal & Gopal-Krishna 1985), or by the inverse Compton catastrophe ($\sim 10^{11} - 10^{12}$ K, Kellermann & Pauliny-Toth 1969). If an appropriate variability timescale, $\tau_{obs}$, is found corresponding to an observed flux variation $\Delta S$ measured at a frequency $\nu$, then (ignoring cosmological effects)

$$T_{B,obs} \propto \Delta S/(\tau_{obs}\nu)^2$$

and

$$\delta = (T_{B,obs}/T_{max})^{1/(3+\alpha)},$$

where $\alpha$ is the spectral index ($S_\nu \propto \nu^{-\alpha}$) (e.g., Teräsranta & Valtaoja 1994). This last method, which actually produces a lower bound to $\delta$, or $\delta_{min}$, has been used quite commonly because it does not require VLBI measurements (e.g., Fantii et al. 1983; Singal & Gopal-Krishna 1985; Teräsranta & Valtaoja 1994).

The foregoing equations (1) and (2) can be combined to solve for $\Gamma$ and $\theta$ of the knot, assuming a cylindrical jet (e.g., Guerra & Daly 1997), in terms of $\beta_{app}$ and $\delta_{min}$,

$$\Gamma = \frac{\beta_{app}^2 + \delta_{min}^2 + 1}{2\delta_{min}},$$ \hfill (3)

and

$$\tan \theta = \frac{2\beta_{app}}{\beta_{pp}^2 + \delta_{min}^2 - 1}. \hfill (4)

Within the context of this last method for estimating $\delta_{min}$, we shall now proceed to quantify how the solutions for $\Gamma$ and $\theta$ (Eqs. 3 and 4) are affected when an allowance is made for the jet’s conical geometry (with a finite full opening angle, $\omega$), which, as mentioned above, can serve as a few degrees on parsec scales.

The procedure for computing the mean flux boosting factor, $\bar{A}$, which is the same as the brightness temperature boosting factor, such that the observed flux $S_{\nu,obs} = \bar{A}S_{\nu,em}$, with $S_{\nu,em}$ the emitted flux, has been described in GDW. Also described there is the way in which the boosting weighted effective apparent speed, $\beta_{app,\nu}$, is found by integrating over the jet cross section. The effective value (that would be observed) of $\delta_c$ is

$$\delta_c = (\bar{A})^{1/(3+\alpha)},$$ \hfill (5)

where we have assumed that the emission is concentrated in the knot; if the emission were from a continuous flow the integer in the exponent in Eq. (5) would be 2 instead of 3. In the following, we have made the common assumption of a flat radio spectrum for the VLBI knots ($\alpha = 0$), though our main conclusions are not sensitive to reasonable deviations from this flatness assumption for core dominated sources.

These effective parameters can now be used to compute the values of $\Gamma_{inf}$ and $\theta_{inf}$ that would be inferred from the standard formulae (3) and (4). These inferred values can then be compared with the actual intrinsic values of $\Gamma$ and $\theta$ adopted for the jet while computing $\delta_c$ and $\beta_{app}$.

3 RESULTS

Taking several combinations of $\Gamma$ and $\omega$ for jets, we plot in Fig. 1 the computed $\theta$-dependences of the values of $\Gamma_{inf}$ that would be inferred from flux variability and VLBI proper motion data in the conventional approach where the angular width of the (parsec-scale) jet is ignored by assuming a cylindrical geometry. Alongside these $\Gamma_{inf}$ curves, we also
show the probabilities, \( p(\theta) \), of observing a jet in a flux limited sample at a viewing angle of \( \theta \). All curves are computed using a grid spacing of 0.01° within the jet’s crosssection.

Following GDW, this probability function, \( p(\theta) \), is given by

\[
p(\theta) \, d\theta \propto \sin \theta A^q(\theta) \, d\theta,
\]

where \( q \approx 3/2 \) is the slope of the integral source counts (Log \( N - \log S \)) at centimetre wavelengths (e.g., Kapahi 1987; Cohen 1989). We also show in Fig. 1 the expectation value, \( \langle \Gamma_{\text{inf}} \rangle \), which is found by weighting the values of \( \Gamma_{\text{inf}}(\theta) \) by \( p(\theta) \).

In Fig. 2 we plot values of \( \theta_{\text{inf}} \) against \( \theta \), along with the expectation values of \( \langle \theta_{\text{inf}} \rangle \), which are also computed for a flux-limited sample using the same \( p(\theta) \) distributions. As expected, for small enough \( \omega \), about 1°, the inferred values of \( \Gamma \) and \( \theta \) are both quite close to the adopted intrinsic values, although even then for very high \( \Gamma \) a difference at the \( \sim 20 \) per cent level for \( \Gamma_{\text{inf}} \) is seen.

The characteristic behaviour of \( \Gamma_{\text{inf}} \) can be segmented into three regimes. For \( \theta \) less than some critical angle, \( \theta_c \approx \omega/2 \), \( \Gamma_{\text{inf}} \) remains essentially constant at a value which can be much smaller than \( \Gamma \). The computed expectation value is dominated by this reduced \( \Gamma_{\text{inf}} \), since for \( \theta > \theta_c \), the probability of viewing such a source, \( p(\theta) \), drops drastically (Fig. 1). Approaching the critical viewing angle from below, a sharp rise in \( \Gamma_{\text{inf}} \) to a value exceeding \( \Gamma \) is found, the excess being more pronounced for larger \( \omega \). Finally, at still larger \( \theta > \theta_c \), \( \Gamma_{\text{inf}} \) declines and asymptotically approaches \( \Gamma \); however, the chance of seeing a source in either of these last two regimes is very small. This behaviour is a basic consequence of the spatial sharpness of the region of strongest Doppler boosting, across which the gradients of \( \beta_{\text{app}} \) and \( A \) can be positively or negatively correlated. Another key factor is how much of this region (whose size is \( \sim 1/\Gamma \)) is encompassed within the jet’s crosssection at a given viewing angle. Clearly, for large \( \Gamma \), this coverage is very sensitively dependent on \( \theta \) when \( \theta \) approaches \( \theta_c \). In particular, as the periphery of the jet’s crosssection crosses over the line-of-sight (\( \theta < \omega/2 \)) the value of \( \beta_{\text{app}} \) averaged over the cross-

\[ \text{Figure 1. Inferred Lorentz factors, } \Gamma_{\text{inf}}, \text{ (solid curves) and flux-limited sample probability functions, } p(\theta), \text{ (dot-dashed curves) against the viewing angle to the jet axis, } \theta. \text{ The top row has } \Gamma = 100, \text{ the middle row, } \Gamma = 50 \text{ and the bottom row, } \Gamma = 10. \text{ Each panel is labelled with the actual value of the full opening angle } \omega. \text{ Also noted with arrows are the expectation values, } \langle \Gamma_{\text{inf}} \rangle. \]
section will decrease from a value which was comparable to
Γ (and will become 0 when θ = 0), leading to a fall in Γinf
(Eq. 3).

In Fig. 2 the most noticeable feature is the sharp drop in θinf as θ approaches θc from above. This can also be
understood because at this viewing angle the jet’s cross-
section begins to encompass the narrow region close to the
line-of-sight (θ < 1/Γ) where the Doppler boosting is most
extreme. Thus the flux-weighted viewing angle of the beam,
θinf, drops to very small values. For θ > θc the values of
θinf begin to approach θ itself, and once θ > ω the inferred
viewing angle barely differs from the actual one. The expec-
tation values, ⟨θinf⟩, are found to be nearly independent of ω,
and scale approximately (in radians) as 0.5/Γ, somewhat less
than the most probable value for ω = 0, 1/Γ. To summarize,
the standard procedure of estimating Γ and θ from δ values
obtained from flux variability measurements can grossly un-
derestimate their values if the jets are highly relativistic
and have modest opening angles. Often the standard procedure
may yield implausibly precise alignment (θinf ≪ 1◦) even
when the true viewing angle (to the axis of the jet) is a few
degrees.

Another important parameter that may also be sub-
stantially underestimated is the width of the VLBI knots,
thereby making the pc-scale jet appear much better colli-
mated than it really is. For instance, Jorstad et al. (2005)
find that most of their 7mm VLBA observations indicate full
de-projected opening angles of only 1–4◦. However, these
could be underestimated, since for an ultrarelativistic jet
the Doppler boosting is extremely pronounced for the por-
tion of the jet’s cross-section which lies within an angle of
∼ 1/Γ to the line-of-sight. To demonstrate this quantita-
tively, in we plot in Fig. 3 the FWHM, FWQM, and full-
width at one-tenth maximum (FW(0.1)M) of the Doppler
boosting distribution, thereby providing estimates for the
de-projected opening angles, ωinf, that are expected to be
inferred from VLBI data. These are computed in an approx-
imation only valid for ultrarelativistic jets when ω > 1/Γ.
We have plotted these results only for ω = 5◦ since the corre-
sponding plots for ω = 10◦ are nearly the same. As expected,
these inferred opening angles basically scale with Γ−1. Note
that these computed ωinf values are slight underestimates,
as they should be convolved with the beaming pattern of
each element of the jet’s cross-section, which would widen
the FWHM to a minimum value of 2/Γ.

4 DISCUSSION AND CONCLUSIONS

In our earlier paper (GDW) we quantitatively checked the
consistency with the VLBI proper motion data of the hy-
pothesis that, on the parsec-scale, many blazar jets are ex-
tremely relativistic ($\Gamma > 30$) and have full opening angles of several degrees. In this paper we argue for the consistency of such high Lorentz factors with observations of flux variability, by showing that standard analysis of such data often leads to underestimation of $\Gamma$ and $\theta$ if the conical shape of the jets is not properly taken into account. Ultrarelativistic jets have been independently inferred recently from $\gamma$-ray variability (e.g. Ghisellini et al. 2005; Krawczynski, Coppi & Aharonian 2004) and modelling of TeV blazar spectra (e.g. Krawczynski 2004, and references therein) as well as rapid radio variations (Begelman, Ergun & Rees 2005, and references therein) and very high $T_B > 10^{13}$ K deduced from space VLBI (e.g., Horiuchi et al. 2004; Kovalev et al. 2005). In our earlier work we showed that the frequently observed subluminal or mildly superluminal apparent motions of TeV blazar VLBI knots can be reconciled with ultrarelativistic jets provided they have a roughly conical geometry with modest full opening angles, $\omega$, of $5 \times 10^{-5}$ on the parsec scale.

Here we show that such characterizations of jets can also explain the smaller values of $\Gamma$ and $\theta$ that are usually inferred by combining the observations of apparent motions of VLBI knots with their radio flux variability under the assumption of a cylindrical jet ($\omega \rightarrow 0^\circ$). We note from Fig. 1 that the expectation values of the inferred $\Gamma$ for $\Gamma = 50$ to 100 and $\omega = 5^\circ$ to $10^\circ$, are in the range 17–34. While these values for $\Gamma_{\text{inf}}$ are still roughly twice those typically inferred from the analysis of variability data of blazars (e.g. Teräsranta & Valtaoja 1994), since the latter estimates of Lorentz factor are only lower limits, via a $\delta_{\text{min}}$, there need not be any contradiction. Our values for $\Gamma_{\text{inf}}$ are not much above those found by Jorstad et al. (2005) who use a modified variability technique which compares the timescale of decline of flux density with the light travel time across the emitting region. Likewise, we have shown that the tendency of VLBI jets to appear quite well collimated (with $\omega_{\text{inf}} \sim 2^\circ$, Jorstad et al. 2005) can also be reconciled with our assumption of conical jets with opening angles which are actually a few times larger.

For TeV blazars, two key arguments made for strong deceleration of jets between the TeV emitting sub-parsec scale to the radio emitting parsec scale are: (i) The frequently observed mildly relativistic or sub-relativistic motion of the VLBI knots (e.g., Piner & Edwards 2004); (ii) the problem with FR I–BL Lac unification, since $\Gamma > 10$ would grossly overpredict the number of FR I galaxies above a given limiting flux (e.g., Hardcastle et al. 2003). We stress that the conical jet geometry could resolve both of these problems for non-decelerated flows (see below).

Piner & Edwards (2004) have reported that the few known TeV blazar display at most mildly superluminal VLBI knots, in statistically significant contrast to normal blazars, which show both mildly and truly superluminal radio knots (Jorstad et al. 2001a,b). To address this intriguing absence of relativistic shocks in TeV blazars, Piner & Edwards considered the alternative possibility that the apparent subluminal motion in TeV blazars could be due to a closer alignment of the jet to the line-of-sight. However, since the required angle is $\leq 1^\circ$, they discount this scenario on statistical grounds and instead favoured the possibility that the jets actually decelerate to modest bulk Lorentz factors between sub-parsec and parsec scales. (However, this raises the question: what renders the deceleration mecha-

nism less effective in the case of normal blazars?) In GDW we have shown that the slow apparent motion of VLBI knots can be an artefact resulting from the finite opening angle of the jet. From our present analysis (Sect. 3) it is further evident that the jet misalignment angle required to account for mildly relativistic or non-relativistic apparent motion can be as large as a few degrees (instead of $<1$ degree). This is statistically much more plausible, so the hypothesis of the jet’s rapid deceleration is not necessary on this account.

The second difficulty, related to the parent population of BL Lacs, can be addressed by recalling that the beaming angle for an ultra-relativistic conical jet would at least equal its opening angle (even for an idealized case when the bulk motion of the radiating plasmons within the jet is ballistic, i.e., purely along straight trajectories). Thus, the effective beaming angle of an ultrarelativistic jet being considered here ($\Gamma \sim 50$, $\psi \sim 10^\circ$), could easily correspond to that expected for a $\Gamma \sim 5$ cylindrical jet and hence be consistent with the FR I–BL Lac unification scheme (e.g., Urry & Padovani 1991; Hardcastle et al. 2003). For misalignment angles larger than the cone angle, it is conceivable that appreciable flux is received from a core component of the type detected in the nearby radio galaxy M87, which is non-varying (Kovalev et al. 2005) and hence probably at most mildly beamed.

In this context, it may be worthwhile to recall the cases of two blazars where modelling of the broadband spectra has yielded no evidence for deceleration of highly relativistic jets from subparsec scales to distances of hundreds of kiloparsecs (Tavecchio et al., 2004).

We note that our aim in the present study is not to reject the possibility of a drastic deceleration taking place between sub-parsec and parsec scales, accompanied with the formation of spine-sheath velocity structure in parsec scale jets, as inferred, e.g., by Chiaberge et al. (2000) (also, Marscher 1999; Trussoni et al. 2003). We merely seek to question the generality of this scenario as well as the underlying tacit assumption that the process responsible for the creation of radio emitting knots (e.g., some instability whose source lies at the base of the jet) operates almost exclusively within the (slower) sheath region. We have argued that even when radio knots form within the putative ultrarelativistic spine, this can be in accord with the observed subluminal motions without resorting to the assumption of a very close jet alignment, which is highly implausible. Still, one expects the jets in TeV blazars to be fairly well directed towards us, owing to the dependence of the $\gamma$–ray flux on a high power of $\delta$ (cf. Piner & Edwards 2004). Our model does predict that a small fraction of TeV blazars should show strongly superluminal motions (GDW). However, in light of the modest number of such sources known so far, and the smaller number which have had their structure mapped frequently with VLBI, the fact that no such apparent velocities have been found is to be expected. As more such blazars are discovered by HESS, MAGIC and VERITAS, the sample on which the VLBI studies can be performed should become large enough to test this prediction.

The situation appears to be much less intriguing for normal blazars, as seen from two large samples observed at cm/mm wavelengths using the VLBA (Jorstad et al. 2005; Kovalev et al. 2005). While a sizable fraction of these blazars does exhibit subluminal motion, Lorentz factors estimated
for the vast majority fall within the range 5 to 40 (e.g., Jorstad et al. 2005). Similarly large values of $\Gamma \gtrsim 30$ have been estimated in the past from VLBI monitoring of a few well known blazars (e.g., Fujisawa et al. 1999). Note that the occurrence of subluminal radio knots in a small fraction of normal blazars is fully consistent with the present model invoking modestly misaligned ultrarelativistic conical jets (GDW).

We note that recent VLBI studies have also revealed several cases of ultra-bright radio knots, with $T_B$ extending up to (at least) $5 \times 10^{13}$ K (Kovalev et al. 2005; Horiiuchi et al. 2004). In addition, fairly robust estimates of angular sizes have been made for a few blazars from their intra-day flux variations at centimetre wavelengths, yielding brightness temperatures of $5 \times 10^{13}$ K or more (Rickett et al. 2002; Macquart & de Bruyn 2006), which cannot be understood in terms of a combination of modest Lorentz factors and refractive interstellar scintillation. As discussed by these authors, interpretation of at least these ultra-bright sources in terms of steady incoherent synchrotron radiation would still demand extremely large bulk Lorentz factors ($\Gamma \gtrsim 100$). The only way to avoid such large $\Gamma$ values appears to be to invoke some coherent radiation mechanism (e.g., Begelman et al. 2005).

One disadvantage of the ultrarelativistic scenario is a decreased efficiency in the conversion of bulk energy to radiated energy (e.g. Begelman, Rees & Sikora 1994); however, this problem is, on average, no more severe for this conical jet picture than it is for a cylindrical jet with the same $\Gamma$. If one compares two jet knots with identical intrinsic luminosities and emissivities, both with $\Gamma = 50$, the ratio of the observed flux from a jet with full opening angle $\omega = 5^\circ$ to that observed from the cylindrical equivalent clearly depends on the (actual) viewing angle, but the range is not very dramatic. This ratio, $A/\delta^3$, is: 0.17 at $\theta = 0.5^\circ$; 0.54 at $\theta = 1.0^\circ$; 5.09 at $\theta = 2.0^\circ$; 2.93 at $\theta = 5.0^\circ$; and is 1.0 for $\theta \geq 15^\circ$. We have already shown (GDW) that the limit on total jet power set by a comparison between the Eddington luminosity and the inferred bolometric luminosity is not a significant constraint.

The presence of ultrarelativistic bulk motion in the VLBI jets, as argued here to be in accord with a variety of observations, have other interesting observational implications. For instance, the deprojected length of jets as well as the radio lobe separation could be substantially overestimated, since the actual viewing angle is often much larger than the $\theta_{\text{inf}}$ inferred by assuming the jet to be cylindrical (Sect. 3). The substantial reduction in the deprojected lengths of the jets, as argued here, has important ramifications in several contexts, one of which is the origin of optical and x-ray emission associated with radio hot spots. (e.g., Gopal-Krishna et al. 2001; Prieto et al. 2002; Brunetti et al. 2003). Further, the possibility of substantial underestimation of viewing angle can ease the disconcerting inference that exceptionally large radio lobes are associated with several prominent blazars (Schilizzi & de Bruyn 1983).

In addition, the inverse Compton boosting of the UV photons from the accretion disk by $\Gamma \sim 10$ jets would lead to an additional contribution to the SED in the soft x-ray band, the so-called “Sikora bump”; its absence in blazar spectra has been used to argue against lepton dominated relativistic jets on the parsec-scale (Sikora & Madejski 2000). On the other hand, for $\Gamma \sim 50 - 100$ jets the corresponding bump would be pushed up to very hard x-rays where it is presently much more difficult to detect, and so the lack of the soft x-ray Sikora bump does not currently constrain such models. We finally note that ultrarelativistic jets containing shocks with finite opening angles, similar to those discussed here for blazars, are usually invoked in modelling the afterglows from Gamma Ray Bursts (e.g., Sari, Piran & Halpern 1999; Panaitescu & Kumar 2000; Frail et al. 2001). Thus the plausibility of very high Lorentz factors of blazar jets hints at an underlying similarity between the jets of GRBs and blazars.

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Bulk Motion of Ultrarelativistic Conical Blazar Jets

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