Accelerated expansion of the universe driven by tachyonic matter

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It is an accepted practice in cosmology to invoke a scalar field with a potential \( V(\phi) \) when the observed evolution of the universe cannot be reconciled with theoretical prejudices. Since one function degree of freedom in the expansion factor \( a(t) \) can be traded off for the function \( V(\phi) \), it is always possible to find a scalar field potential which will reproduce a given evolution. I provide a recipe for determining \( V(\phi) \) from \( a(t) \) in two cases: (i) a normal scalar field with the Lagrangian \( \mathcal{L} = (1/2)\dot{\phi}^2 - V(\phi) \) used in quintessence or dark energy models; (ii) a tachyonic field with the Lagrangian \( \mathcal{L} = -V(\phi)(1-\partial_t^2 \phi^2)^{1/2} \), motivated by recent string theoretic results. In the latter case, it is possible to have accelerated expansion of the universe during the late phase in certain cases.

I. INTRODUCTION

The stress tensor \( T^\mu_\nu \) for any source term in a Friedmann universe, described by an expansion factor \( a(t) \), must have the form \( T^\mu_\nu(t) = \text{dia} \left[ \rho(t), -p(t), -p(t), -p(t) \right] \). Given an equation of state which specifies \( p \) as a function of \( \rho \), we will be left with two degrees of freedom \( a(t) \) and \( p(t) \) which can be determined by two independent Einstein’s equations for the Friedmann model. The situation, however, is slightly different if the source is described by an “adjustable function”, as in the case of scalar fields. If, for example, the source is described by a scalar field with a Lagrangian \( \mathcal{L} = (1/2)\partial_t \phi \partial^2 \phi - V(\phi) \), then it is possible to choose \( V(\phi) \) in order to have a specific evolution for the universe. Given any \( a(t) \) it is usually possible to obtain a \( V(\phi) \) such that it results in a consistent cosmological evolution. In fact, this can be achieved even in the presence of other energy densities in the universe (such as matter, radiation, etc.) in addition to the scalar field. This should not be surprising, since the existence of a free function \( V(\phi) \) allows a trade off with another function \( a(t) \).

Recently, it has been suggested that [1] the tachyonic condensate in a class of string theories can be described by an effective scalar field with a Lagrangian of the form \( \mathcal{L} = -V(\phi)(1-\partial_t^2 \phi^2)^{1/2} \). The evolution of this condensate can have cosmological significance which may be worth exploring [2]. (Scalar field models with a nonquadratic scalar kinetic term were studied on phenomenological grounds earlier in [3]; time dependent solutions to the tachyonic field were studied earlier in [4].) Since this Lagrangian also has a potential function \( V(\phi) \), it seems reasonable to expect that any form of cosmological evolution [that is, any \( a(t) \)] can be obtained with the tachyonic field as the source by choosing \( V(\phi) \) “suitably.” It turns out that this is indeed true.

I will outline a recipe for constructing \( V(\phi) \) given a particular form of evolution for the universe \( a(t) \) in the two cases (normal scalar field and tachyonic field) mentioned above. The first case corresponds to quintessence or dark energy models and has been a favorite pastime of the cosmologists in the past several years (for a sample of references, see [5]). In this case, there is very little (independent) constraint on \( V(\phi) \) and hence it is not possible to evaluate the relative merits of different choices for \( V(\phi) \). In the case of tachyonic scalar field, there are some constraints on the form of \( V(\phi) \), especially on the asymptotic behavior, which could rule out a certain class of cosmological models. It is nevertheless possible to construct several interesting models satisfying the asymptotic constraints on \( V(\phi) \). In particular, it may be possible to have a rapidly accelerated phase of expansion for the universe at late times which seems to have some observational support.

II. RECIPE FOR THE SCALAR FIELD POTENTIAL

Consider a \( k=0 \) universe with a normal scalar field having a potential \( V(\phi) \) as the source. We assume that the evolution of the universe is already specified so that \( a(t), \, H(t)=(\dot{a}/a) \ldots \) etc. are known functions of time and we need to determine \( V(\phi) \) such that Friedmann equations

\[
\frac{\ddot{a}}{a} = \frac{8 \pi G}{3} \rho - \frac{4 \pi G}{3} (p+3p) \tag{1}
\]

as well as the equation of motion for the scalar field

\[
\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi} \tag{2}
\]

are satisfied. (Of course, only two of these three equations are independent when the universe is driven by a single source.) In a Friedmann universe, \( \phi(t,x) = \phi(t) \) and the energy density and pressure of the scalar field is given by

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{3}
\]

It is convenient to define a time dependent parameter \( w(t) \) by the relation \( w(t) = P_\phi/\rho_\phi \). The equation of motion for the scalar field, written in the form \( d(\rho a^3) = -w \rho d(a^3) \), can be integrated to give \( \rho_\phi/\rho_\phi = -3H(t \omega(t)) \).
+w). The Friedmann equation, on the other hand, gives $\rho_\phi \approx H^2$ so that $(\rho_\phi/\rho_{\phi}) = 2(H/H)$. Combining the two relations we get

$$1 + w(t) = -\frac{2}{3} \frac{H}{H^2}$$

(4)

thereby determining $w(t)$. (Note that we have not used the specific form of the source so far; so, this equation will be satisfied by any source in a FRW model.) From the definition of $w$ and Eq. (3), it follows that $\phi^2/2V = (1 + w)(1 - w)^{-1} = f(t)$, say. Writing this as $\phi^2 = 2fV$, differentiating with respect to time, and using Eq. (2) we find that

$$\frac{V}{V} = \frac{-\dot{f} + 6Hf}{1 + f}.$$  

(5)

Integrating this equation and using the definition of $f(t)$ and Eq. (4) we get

$$V(t) = \frac{3H^2}{8\pi G} \left[ 1 + \frac{\dot{H}}{3H^2} \right].$$  

(6)

Substituting back in the relation $\phi^2 = 2fV$, we can determine $\phi(t)$ to be

$$\phi(t) = \int dt \left[ -\frac{\dot{H}}{4\pi G} \right]^{1/2}.$$  

(7)

Equations (6) and (7) completely solve the problem of finding a potential $V(\phi)$ which will lead to a given $a(t)$. These equations determine $\phi(t)$ and $V(t)$ in terms of $a(t)$ thereby implicitly determining $V(\phi)$.

In fact, the same method works even when matter other than a scalar field with some known energy density $\rho_{\text{known}}(t)$ is present in the universe. In this case, Eqs. (6) and (7) generalize to

$$V(t) = \frac{1}{16\pi G} H(1 - Q) \left[ 6H + \frac{2\dot{H}}{H} - \frac{Q}{1 - Q} \right],$$  

(8)

$$\phi(t) = \int dt \left[ \frac{H(1 - Q)}{8\pi G} \right]^{1/2} \left[ \frac{\dot{Q}}{1 - Q} - \frac{2\dot{H}}{H} \right]^{1/2},$$  

(9)

where $Q(t) = [8\pi G \rho_{\text{known}}(t)/3H^2(t)]$. Since $\phi$ has to be real, one can invoke a scalar field in this manner only when the terms within the square brackets in Eq. (9) or Eq. (7) are positive. [For example, Eq. (7) shows that if the scalar field is the only source, then $\dot{H}$ must be negative.] In practice, this is not a serious restriction to model building.

As an example of using Eqs. (6) and (7), let us consider a universe in which $a(t) = a_0 t^n$. Elementary algebra now gives the potential to be of the form

$$V(\phi) = V_0 \exp \left( -\sqrt{\frac{2}{n M_P}} \phi \right).$$  

(10)

where $V_0$ and $n$ are constants and $M_P^2 = 1/8\pi G$. The corresponding evolution of $\phi(t)$ is given by

$$\frac{\phi(t)}{M_P} = \sqrt{2n\ln \left( \sqrt{\frac{V_0}{n(3n - 1)/M_P^2}} t \right)}.$$  

(11)

As a second example, consider an evolution of the form

$$a(t) \propto \exp(\alpha t),\quad f = \frac{\beta}{4 + \beta},$$  

$$0 \leq f < 1,\quad \alpha > 0.$$  

(12)

In this case, we can determine the potential to be

$$V(\phi) \propto \left( \frac{\phi}{M_P} \right)^{-\beta} \left[ 1 - \frac{\beta^2 M_P^2}{6} \right],$$  

(13)

where $\beta$ is a constant. The two potentials described above have been used extensively in inflationary models.

In fact, virtually all other potentials used in quintessence or dark energy models for the universe can be obtained by the recipe given above. Since one seldom worries seriously about the microscopic origin of $V(\phi)$ in these models, it may be mathematically more convenient to choose one’s favorite cosmological evolution in terms of $a(t)$ and then construct $V(\phi)$ and study its properties.

### III. RECIPE FOR TACHYONIC POTENTIAL

Consider next a universe with a given $a(t)$ and a tachyonic source with the Lagrangian $\mathcal{L} = -V(\phi)[1 - \partial_\mu \phi \partial^\mu \phi]^{1/2}$. When $\phi = \phi(t)$, the energy density and pressure are given by

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},\quad p = -V \sqrt{1 - \dot{\phi}^2}.$$  

(14)

In this case, the “reverse engineering” to determine $V(\phi)$ from $a(t)$ is almost trivial. For any source with a parameter $w(t)$, we must have

$$\frac{\dot{\rho}}{\rho} = -3H(t)(1 + w) = \frac{2H}{H}$$  

(15)

leading to Eq. (4). On the other hand, for the tachyonic model, $p(t)/\rho(t) = w(t) = \dot{\phi}^2 - 1$. Combining these, we can determine $\dot{\phi}^2 = -(2/3)(\dot{H}/H^2) = (2/3)(dH/dt)$ in terms of $H$ and obtain

$$\phi(t) = \int dt \left( \frac{2H}{3H^2} \right)^{1/2}. $$  

(16)

Multiplying the two equations in Eq. (14) and using Eq. (4) and the Friedman equation, we get
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the late phases in the evolution of the universe, like, for example, as some kind of dark matter.

An easy way out is to use the solutions in the early universe. The solutions I have obtained, of course, could also be used to provide a power law inflation in the very early universe if \( t \rightarrow \infty \) is interpreted merely as \( t \simeq t_{\text{Planck}} \). It may be easier to have nonvanishing pressure for the tachyonic condensate for \( t \simeq t_{\text{Planck}} \) than during the current epochs of the universe. (I think it would be nicer if string theory could provide an effective cosmological constant at the current epoch—which has some observational support—rather than merely provide yet another inflaton field. But the procedure outlined here can be used to construct a wide class of inflationary solutions as well.)

If, on the other hand, string theory demands \( \phi(\infty) = 1 \) asymptotically, then \( V \rightarrow 0 \) asymptotically in such a manner as to give finite \( \rho \) and zero pressure. The tachyonic condensate can then contribute, say, \( \Omega = 0.7 \) in the universe and—together with clustered normal matter contributing \( \Omega = 0.3 \)—can lead to a \( \Omega = 1 \) universe. Such a model is an extreme form of mixed dark matter model with a very smoothly distributed component at large scales. It is possible that the model is consistent with CMBR and galaxy clustering data but will contradict the supernova data if it is interpreted as indicating \( \ddot{a} > 0 \) in the recent past. (Because \( \ddot{a} < 0 \) when \( n < 1 \), this model will not be accelerating.) Given the observational uncertainties, it may still be worth studying the consequences of such a model.

Even in this case [with \( \phi(\infty) = 1 \) asymptotically], it may be possible to provide an accelerated expansion for the universe in the recent past if one considers more complicated potentials. For example, one can construct models such that \( n > 1 \) in the redshift range of \( 3 > z > 0 \), say, with the asymptotic regime of \( n = (2/3) \) coming into effect only in the future. The procedure developed in this paper can be used to obtain suitable \( V(\phi) \) which will ensure such a scenario.

All these emphasize the need to understand the constraints on \( V(\phi) \) in the Lagrangian from more fundamental considerations. A possible interpretation which is of relevance to cosmology is suggested in [6].

Results partially overlapping with those in the current paper are also reported in [7] which appeared almost simultaneously with this paper.

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